The Green Energy Transition in a Putty-Clay Model of Capital

Simon Gilchrist* Joseba Martinez† Natalie Rickard§

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Abstract

Achieving the green energy transition requires reducing reliance on the existing stock of fossil-fuel dependent capital and increasing the share of investment in green energy. A salient concern among policymakers is that a carbon tax, which addresses the carbon externality and thereby provides the correct incentives for green investment, can lead to stranded assets — capital that loses value due to climate policy. Standard macroeconomic models minimize this trade-off by assuming that fuel usage for existing capital can be freely adjusted after investment. We address this limitation by embedding within an integrated assessment model a putty-clay framework that explicitly captures the expost irreversibility of capital-fuel ratios. Our analysis highlights important trade-offs between achieving climate goals and mitigating economic costs. Carbon taxes must be 50% higher in our putty-clay model to achieve emissions reductions comparable to standard models, yet the optimal carbon tax is only half as large. This divergence stems from the substantial consumption declines that arise as new investment replaces economically unviable capital. We show that grandfathering existing fossil-fuel capital through targeted tax discounts substantially mitigates the stranded asset problem, lowers short-term economic costs and improves welfare relative to uniform carbon taxation.

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*Department of Economics New York University and NBER. Email: sg40@nyu.edu
†London Business School and CEPR. Email: jmartinez@london.edu
‡London Business School. Email: nrickard@london.edu

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1 Introduction

To prevent the worst effects of climate change, the world must undergo a rapid transition away from fossil fuels and toward green energy technologies. Despite rapid improvements and declines in costs of renewable technology[1], fossil fuels still met nearly 80% of global energy demand in 2023 (IEA WEO, 2024). A crucial barrier to this shift is the pre-existing stock of capital which is fossil-fuel-dependent. The IPCC AR6 highlighted that the majority of potential stranded assets are not the fossil fuels themselves, but the capital – coal-fired power plants and other fossil infrastructure – reliant on fossil fuels. Achieving the most ambitious climate targets could amount effectively to a substantial destruction of capital that was previously productive. This presents a challenging trade-off; when is it best to shut down these coal and gas power plants, and would the resulting near-term economic damage be outweighed by the climate benefits?

The novel contribution of our paper is to study these tradeoffs by embedding a putty-clay technology within an integrated assessment model following the tradition of IAMs proposed by Nordhaus, and further studied by Golosov et al. (2014). Putty-clay technology allows us to explicitly model the effects of capital fixity and irreversibility on the green energy transition. Incorporating these features means that the economic costs of imposing green policies like carbon taxes are significantly higher; initial declines in consumption more than double relative to the standard model. In addition, the transition to greener energy is considerably slowed, as firms and households continue to rely on energy produced using pre-existing dirty capital. Taking as given pre-existing climate goals, carbon taxes need to be as much as 50% larger under this framework. However, larger carbon taxes begin to decrease welfare, given the larger near-term economic costs. As a result, the optimal carbon tax is half as large in the putty-clay framework.

A substantial contribution of this framework is that it provides a coherent definition of stranded assets. We show that the value of dirty capital installed prior to policy implementation falls by over 20% after the imposition of a 75$ carbon tax. One way to alleviate these asset price declines is to grandfather in old capital, by giving discounts on carbon taxes for machines that were installed prior to policy implementation. Moreover, this policy can improve welfare, as it keeps older capital viable and moderates near-term contractions in consumption.

Putty-clay technology allows us to model important features of energy production and the green transition. We show, using data from the US Department of Energy, how energy production has begun to shift towards more energy efficient and greener production technologies. An important component of this shift in the data is the capital composition within the energy sector; much of energy production is generated from long-lasting capital, which is only gradually retired but increasingly underutilised as new technologies are installed. We can capture this age distribution of capital within our framework. The model features a final goods sector that requires energy inputs, which can be produced by either a fossil-fuel burning sector or a clean energy sector. Fossil fuel usage leads to an accumulation of car-

[1] For example, the global weighted average levelised cost of electricity (LCOE) of newly commissioned utility-scale solar PV projects fell by 88% between 2010 and 2021 (World Energy Transitions Outlook, 2023).

bon in the atmosphere, which reduces productivity. The fixed nature of input factors leads endogenously to differential short and long-run elasticities of substitution to input prices, a crucial feature of the response to energy shocks which is hard to capture when capital is treated as fungible. Ageing capital becomes increasingly underutilised and following policy changes, portions of the capital stock become 'stranded assets'.

Using this framework, we study two main experiments: an improvement in green technology calibrated to match the fall in solar panel prices from 2008 onwards, a carbon tax calibrated to approximately \$75 per metric ton of CO^2. Following an increase in green technology, there is a long-run expansion of aggregate output, consumption and investment of approximately 6% relative to trend over 40 years, as the economy benefits both from improved technology and a reduction in the economic damage from the climate externality. There is a substantial shift within the energy sector away from dirty to clean energy; dirty energy production falls by 30%, while clean energy increases by 400%. As a result, fossil fuel usage declines and carbon accumulation slows. Because of the gradual introduction of new technology, the medium-term aggregate dynamics of the putty-clay model broadly coincide with those of the standard model. However, in the short-run there is a greater contraction in dirty energy sector investment and a higher utilisation of pre-existing dirty, polluting machines in the putty-clay model, resulting in relatively more fossil fuel use compared to the standard model.

Carbon taxes are both more effective in reducing emissions, but more damaging for economic outcomes, compared to green technology improvements. Following an imposition of a \$75 carbon tax, output contracts on impact by around 3%, with short-term declines in investment and consumption. Fossil fuel usage declines by nearly 60% in the long run, as energy usage declines and production shifts toward green energy.

There are striking differences in the impact of carbon taxes under a putty-clay model, compared with standard frameworks however. In models with fungible capital, machines can be transformed to reduce their fossil fuel usage, shifting to either more energy efficient or green forms of production. In contrast, in our more realistic framework, dirty machines which rely heavily on fossil fuels are utilised less – many of these assets are mothballed and are in effect sunk, as they are less likely to be profitable under the new policy. Instead, the energy sector has to invest more heavily in both new, more fuel efficient dirty capital, and more green capital. Despite these adaptations, firms with pre-existing dirty machines still require fossil fuels to produce energy, so fuel usage falls more slowly and greater damages from emissions are experienced under the putty-clay model. To achieve the same carbon targets as under the standard model, a 50% larger carbon tax is required. In addition, the impacts on welfare are heavily contrasting across the two models. A carbon tax reduces the externality from climate damage and increases welfare under the standard framework. In contrast, because of the additional cost of transforming the capital stock under the putty-clay framework, carbon taxes greater than 150% begin to look unattractive. Moreover, the optimal tax is half as large.

A prominent policy concern when imposing carbon taxes is the loss in value of fossil-fuel burning assets. Unlike the standard model, our framework provides a meaningful measure of stranded assets – that is, assets that become economically unviable once carbon taxes are imposed. These older dirty machines have a fixed fuel input ratio which makes them less profitable, so their utilisation falls. The value of these machines falls gradually, peaking at a 20% decline by seven years after the shock. This gradual decline is due to the fact that in the near-term, before new fuel efficient machines can be introduced, old dirty machines are still in use. The fact that dirty asset valuations only fall gradually on imposition of a carbon tax – even one which is immediately imposed with prior no announcement – may ameliorate concerns over financial instability resulting from stranded asset valuations dropping sharply.

Finally, we show that grandfathering in old dirty machines by giving a discount on carbon taxes can be welfare improving. This is because dirty energy prices rise in response to the carbon tax, so the profitability of currently utilised inframarginal machines increases. As a result, the near-term drop in consumption is moderated, as less investment is needed and more old dirty machines can continue to be used. This finding raises the possibility that a grandfathered carbon tax may be more politically feasible, as the owners of pre-existing dirty assets experience capital gains rather than losses.

Related literature This paper builds on a long tradition of modelling the interaction between climate and the macroeconomy. Nordhaus (1977, 1991, 1992) pioneered this literature, developing integrated assessment models (IAMs hereafter) which introduce climate blocks and carbon accumulation into macroeconomic models, and use these to assess how policies may mitigate climate change. Golosov et al. (2014) built on this, developing a tractable IAM to assess social costs of carbon and carbon pricing. Numerous contributions to this literature have introduced important elements and dynamics in climate-macro modelling. Acemoglu et al. (2012); Popp (2002), and Hassler et al. (2021) show the importance of improvements in green technology and subsidies for green R&D for a green transition. Cruz and Rossi-Hansberg (2024); Desmet and Rossi-Hansberg (2015) and Krusell and Smith (2022) contribute models which take into account the heterogeneity of climate impacts across space. Weitzman (2009, 2014) and more recently Cai and Lontzek (2019) show how uncertainty and potential fat tails in risks from climate change climate change can justify much higher social costs of carbon. Barnett et al. (2020) extend this work to consider additional key components of uncertainty induced by climate change, including ambiguity over different models and model misspecification; Barnett et al. (2022, 2024) build on this, showing how this impacts social costs of carbon and demonstrating how to decompose uncertainty into different underlying sources. Folini et al. (2024) and Dietz et al. (2021) discuss the calibration of the climate blocks of integrated assessment models; we build on insights from the former to calibrate our model.

Complementary to these modelling efforts, a broad literature explores the economic damage resulting from climate damages and assesses policies to reduce future emissions. Deschênes and Greenstone (2012); Dell et al. (2012) and Burke et al. (2015) are among those who have used local weather variation to assess the impact of climate on output and other economic outcomes. Recent contributions, including Nath et al. (2024) and Bilal and Känzig (2024), suggest that the damage from climate change could be larger than previously appreciated. Pertinent to the policy experiments we explore in this paper, Metcalf and Stock (2023) and Känzig (2023) explore the macroeconomic consequences of carbon taxes, and Aghion et al.

(2016) and Acemoglu et al. (2019) explore green technology improvements. We model the impact on valuation of dirty and green energy firms, stock market effects of different carbon taxation approaches and impacts of subsidies and taxes on green and dirty investment, which relate both to government policies and a large literature which explores private investor preferences for ESG investments and impact of climate policy news (e.g. Engle et al. (2020), Bolton and Kacperczyk (2021) and Berg et al. (2021),Hsu et al. (2023)). Fornaro et al. (2024) explore how supply constraints in dirty energy sectors over a green transition may change the shape of the Phillips curve and generate state-specific inflation, affecting the conduct of monetary policy.

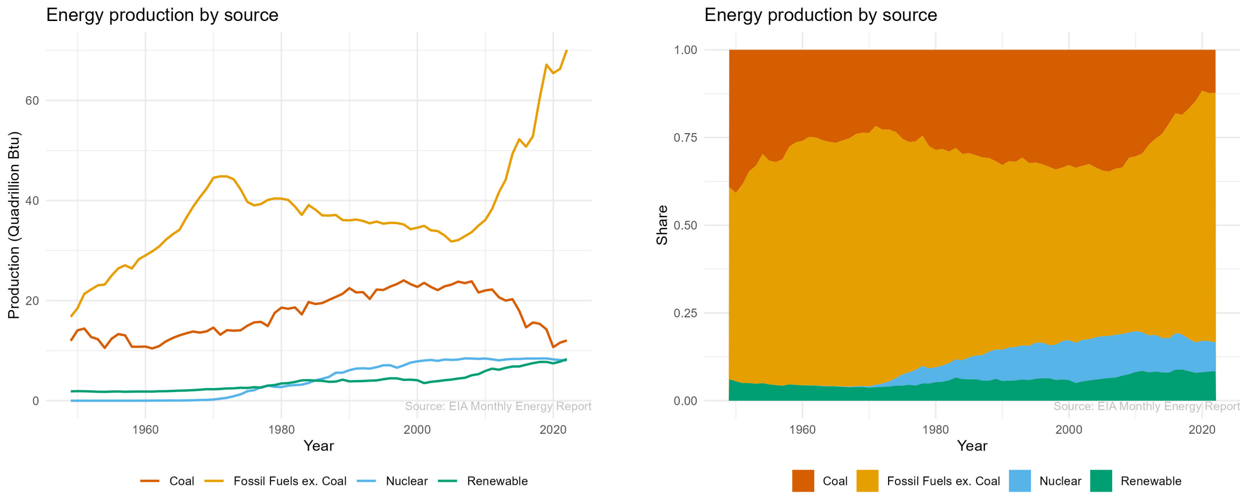
Johansen (1959) introduced the putty-clay framework for modelling capital. In this paper, we build on the approach to putty-clay modelling of Gilchrist and Williams (2000). Relative to other approaches (Atkeson and Kehoe (1999) and Cooley et al. (1995)), this allows tractable modelling of both irreversible investment and variable utilisation of capital, key elements of the putty-clay framework. Using this framework, Gilchrist and Williams (2005) show how increases in uncertainty of the productivity of investment can affect macroeconomic dynamics and Gilchrist and Williams (2004) show how this approach can help understand the post-war economic transitions of Germany and Japan. Hurst et al. (2022) explore how putty-clay technology can drive differential employment effects of minimum wage changes in the short and long run. We follow the approach of Wei (2003, 2013) to modelling fuel and labour as separate variable input factors within the putty-clay framework. Some recent contributions in the climate literature have addressed the importance of accounting for the vintage structure of capital and locked-in investments in particular technologies, including Meng (2022), Lanteri and Rampini (2022) and Hawkins-Pierot and Wagner (2024). Our contribution is to developing a multi-sector IAM with full features of putty-clay technology in each sector.

2 Descriptive Data

This section describes broad trends in energy production and emissions from the use of fossil fuels for the U.S. economy. It also provide a more detailed description of trends in the electricity sector where much of the transition to green energy is occurring. These trends highlight the shifting sources of electricity production as natural gas replaced coal, and more recently, solar and wind electricity exceed natural gas in terms of new investments. All data are obtained from the Department of Energy.

Figure 1 shows the evolution of energy production across four sources: coal, fossil fuels excluding coal, nuclear and renewables over the period 1950-2022. The left panel plots production in energy units (Quadrillion BTUs) while the right panel displays the same information as shares of total energy produced.

Energy production has expanded considerably over the 1950-2022 period with most of this expansion due to an increase in energy production from natural gas which rose from 20 to 70 quadrillion BTUs during this time period. This expansion is especially rapid since the shale boom in the mid-2000s. In contrast, energy production from coal rose gradually from 1950 to 2000 and has since fallen in absolute terms. Nuclear energy expanded in the pre-2000



(a) Levels of energy production

(b) Shares of energy production

Figure 1: Energy production, by fuel source

period but has remained relatively flat since then. Finally renewables have grown gradually in the early part of the sample and exhibit a rapid expansion since 2000.

These trends are seen more clearly in the energy shares plotted in the lower panel of Figure 1. In the early 2000s, coal accounted for thirty percent of total energy production. Electricity produced from coal has declined significantly as a share and now accounts for only twelve percent. Energy production from fossil-fuels excluding coal has remained relatively stable as a share of total production and currently accounts for seventy percent of total energy. Nuclear energy and renewables combined have risen in importance over time and now account for 18 percent of total energy production with roughly equal shares at the end of the sample period.

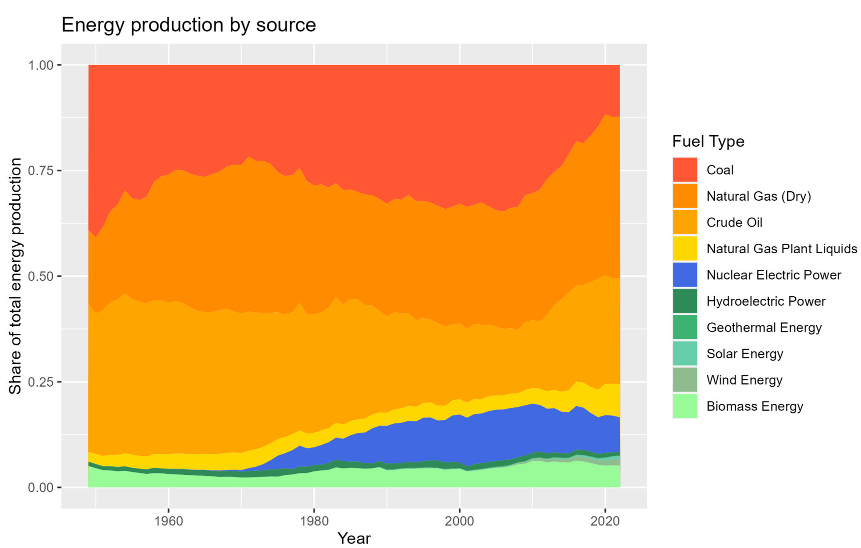


Figure 2: Shares of energy production in detail

Figure 2 provides a more detailed breakdown of energy production by source. In particular, renewable energy is broken out by hydro, geothermal, solar, wind and biomass. Among renewables, biomass accounts for the largest share of energy production. Solar and wind are a small but growing fraction of the total.

Figure 3 shows carbon emissions by fossil fuel source both in levels measured in million metric tons and as a share of total emissions. Total emissions from petroleum has remained relatively flat over time despite the expansion in petroleum use. Emissions from coal rose steadily with production and have now declined while emissions from gas have risen over time.

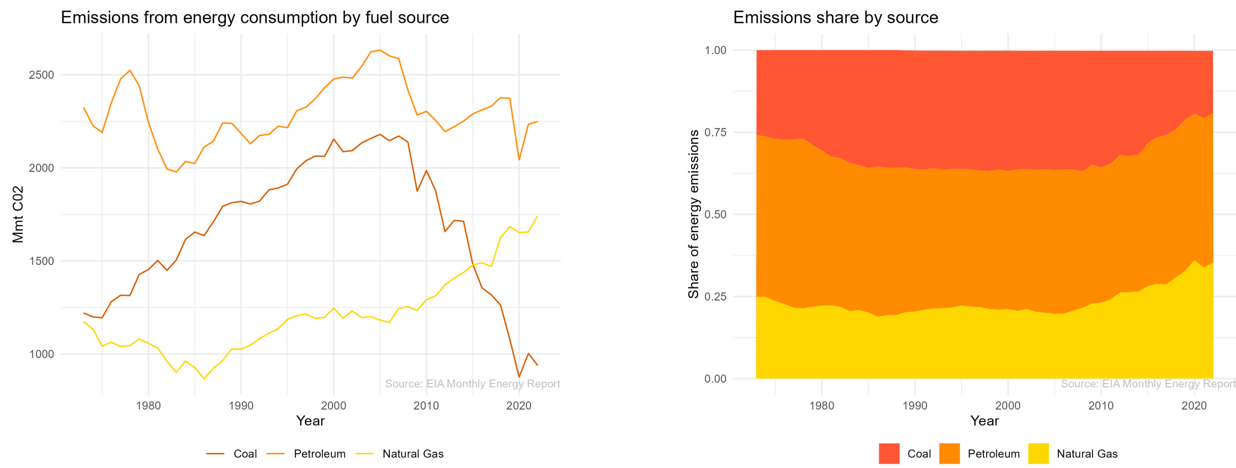


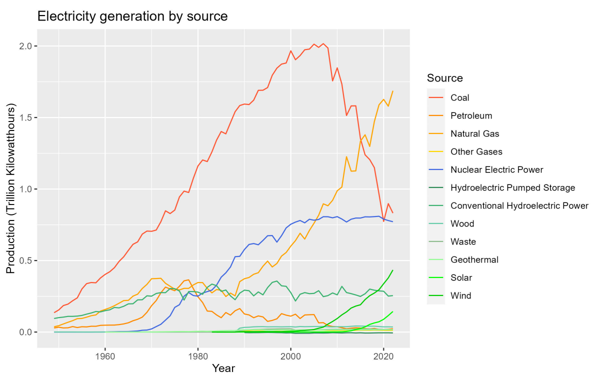
Figure 3: Emissions from energy consumption, by fuel source

The emission shares are displayed in the lower panel of Figure 3. Emission shares differ from production shares due to the fact that fossil fuel emissions differ across fuel types. Emissions from coal are nearly twice as large as emissions from natural gas. Thus, the reduction in coal-powered electricity in favour of gas has had a sizeable effect on total emissions.

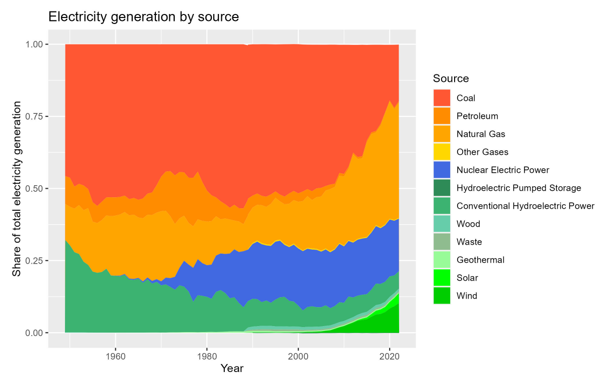
To see clearly the transformation from coal to natural gas along with the nascent adoption of green energy it is useful to focus on electricity production. Electricity production accounts for 34 percent of total energy production in 2022. The next largest sector, transportation, accounts for 27 percent of total energy production. Industrial activity which also produces electricity or heat from fossil fuel sources is the third largest sector and accounts for 24 percent. The importance of the electricity sector provides further motivation for describing additional details of its transformation over time. Moreover the forces that drive transformation in the electricity sector, namely technological change embodied in capital combined with technological lock-in due to the irreversibility of existing capital choices are also highly relevant for the industrial and transportation sectors.

Figure 4 provides the breakdown of electricity generation by source over the period 1950-2022. The left panel displays the total quantity of electricity measured in trillion kilowatt hours. Again one can clearly see the expansion of electricity generated by coal through 2000 and the contraction in electricity produced from coal that followed. One can also clearly see the rapid expansion in electricity generated from natural gas. The other categories of interest are hydro which has remained stable since 1980 and nuclear which has remained stable since 2000 following a rapid expansion over the 1970-2000 period. Finally electricity produced from wind and more recently solar were negligible before 2005 but have expanded rapidly since then.

In terms of shares, the electricity generated from fossil fuels has fallen from 75 percent to 60 percent of total electricity generation in the period 1980-2022. The rapid expansion in wind and solar implies that these two green energy sources now account for twelve percent of total electricity production, a share comparable to the electricity produced by nuclear energy.



(a) Absolute electricity generation by fuel source



(b) Share of electricity generation by fuel source

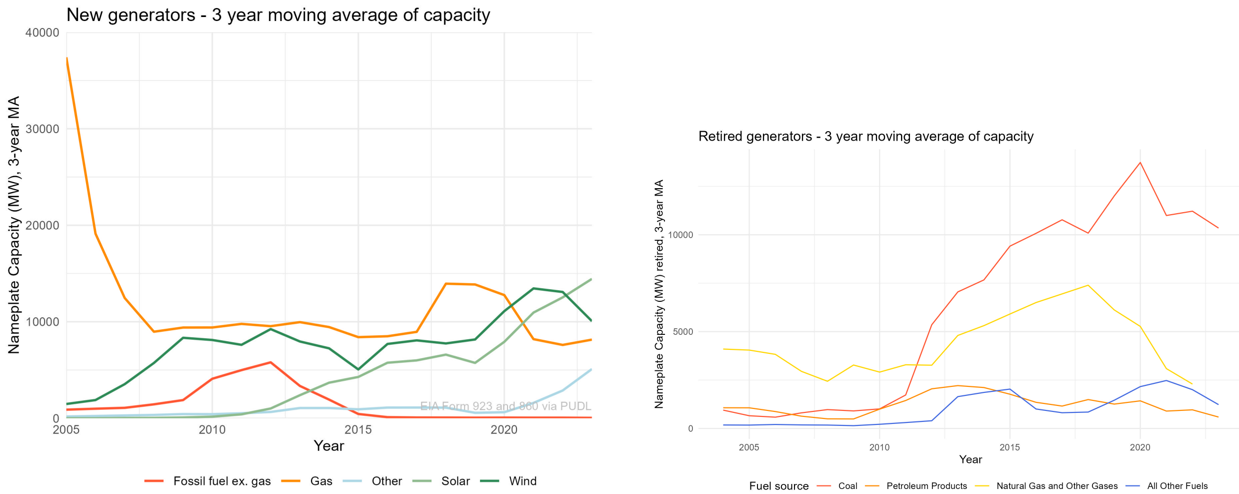
Figure 4: Electricity generation, by fuel source

The patterns of expansion in electricity produced by gas, wind, and solar, and the contraction in electricity produced by coal, can also be seen in the patterns of construction and retirement of electricity generators over time shown in Figure 5, which plots three-year moving averages of the nameplate capacity of new investment (Panel a) and retirement (Panel b) of electricity according to energy source from 2005 to 2020.

New investment in gas generators is highest at the beginning of this sample and has remained fairly constant at 10,000 megawatt-hours since that period. New investment in wind and now solar exceed these amounts by the end of the sample period, while nearly all other sources are minimal. Thus, the electricity sector is currently installing twice as much capacity in green energy relative to dirty energy.

As shown in Panel (b) of Figure 5, simultaneous with the expansion of gas, wind, and solar, there is a significant amount of retirement in coal-powered generators. Thus, electricity capacity from coal contracts. To a lesser degree, there has also been an increase in the retirement of gas-powered generators. The retirement of coal generators is consistent with the overall reduction in electricity generated from coal. Given the ongoing pattern of new investments, the retirement of existing natural-gas electricity generators likely reflects the expansion of newer, more efficient gas-generated electricity plants.

In addition to new construction and retirements, capacity utilization serves as an important margin of adjustment in the electricity sector. Utilization rates decline with age across nearly



(a) Construction of electricity generators

(b) Retirement of electricity generators

Figure 5: Construction and retirement of electricity generators, by fuel source

all types of electricity plants (the two exceptions are nuclear and hydro, where utilization is not an active choice) and also may fluctuate with fuel prices.

Figure 6 displays the variability in utilization rates across generators grouped into plants that are less than forty years in age and plants that are greater than forty years in age. This figure highlights that there is considerable variation in utilization rates across plants and that newer plants are likely to be utilized more intensively

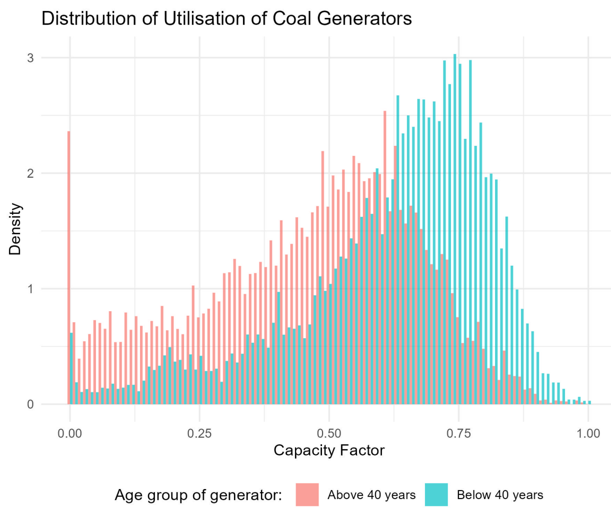


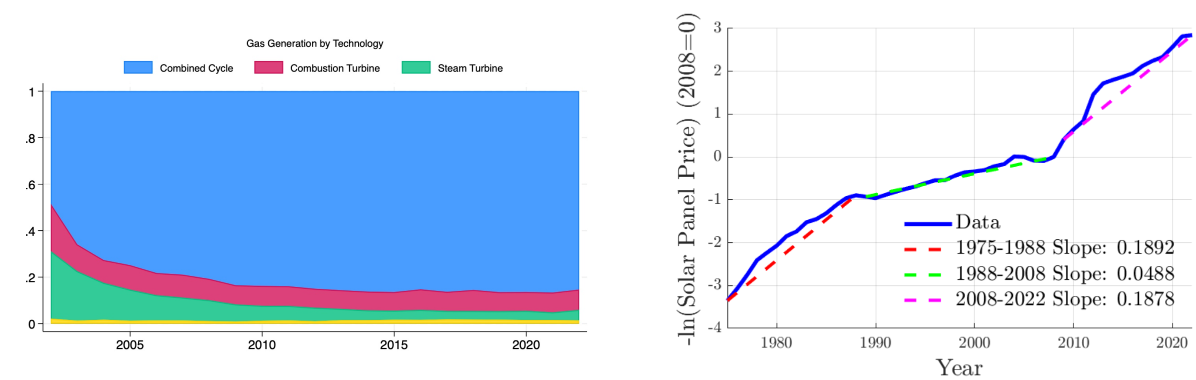
Figure 6: Utilisation distribution for coal generators, by age

Underlying the expansion and contraction of these various source of electricity production is the adoption of new technologies through the purchase of new capacity that replaces existing production. The adoption may occur because of the introduction of a new production method embodied in capital, or because prices for capital that embody the new technology fall over time.

Within the dirty electricity sector, the primary source of technological change is the adoption of more fuel-efficient gas technologies to produce electricity. Early gas-powered electricity plants relied on relatively simple steam turbines that burn gas to heat steam and generate electricity. These technologies have a typical thermal efficiency of 30-40 percent. The combined-cycle gas-powered electricity plants recapture heat exhaust and achieve efficiencies closer to 60 percent.

Figure 7 shows the transformation of gas production and solar technology improvements since 2000. Panel (a) illustrates electricity production by gas technology. At the start of this period, combined cycle generators accounted for fifty percent of gas-generated electricity. By the end of the sample period, they account for 85 percent. This expansion occurred as combined-cycle generators replaced the capacity of steam turbines that were either retired or used less intensively.

Panel (b) shows the declining price of solar panels, measured by the log-inverse price, highlighting increased efficiency in solar power generation. Solar efficiency grew rapidly (20 percent per year) from 1975-1988, somewhat more slowly (5 percent per year) from 1988-2008, and rapidly again since 2008 (approximately 20 percent yearly). This rapid growth in solar technology coincides with increased investment in solar capacity since 2008.



(a) Electricity Production by Gas Technology

(b) Technological Change in Solar Panels

Figure 7: Transformation of Gas Production and Solar Technology

These facts motivate the putty-clay model described below. In particular, the putty-clay model allows for meaningful distinctions between capital vintages and creation of new capacity through investments embodying the most efficient technology. It also allows for active utilization margins that vary with plant age. Finally, the putty-clay framework captures the notion that short-run elasticities of substitution to input prices are substantially below long-run elasticities reflecting the process of building new capacity.

3 Model Description

The model is an integrated assessment model that incorporates the putty-clay technology developed in Gilchrist and Williams (2000). The model has a final goods sector along with dirty and green energy sectors. The final goods sector combines energy, labor and capital to produce final goods. As in Wei (2003), dirty energy uses fossil fuel, labor, and capital as inputs while clean energy uses only capital and labour. These two energy sources are imperfect substitutes in the production of the energy input that is used by the final goods sector.

Within each sector, output is produced using putty-clay technology. Capital goods take the form of machines that are vintage-specific. Without loss of generality, each machine is normalized to employ one unit of labor at maximum capacity. Machines are long-lived and assumed to fully depreciate in any given period according to a Poisson process. The putty-clay technology implies that capital-to-labor and fuel-to-labor ratios are chosen in advance and fixed for the machine's life. Once built, machines are fully utilized as long as the revenue obtained from the machine exceeds the operating cost, which combines fuel and labor costs. All machines within a vintage are identical ex-ante, but are subject to idiosyncratic shocks that determine their productivity once installed. The realized level of productivity is assumed to be permanent and thus embodied in the machine. New machines embody new technologies that improve exogenously over time. Machine productivity may also increase because of growth in disembodied technology. As productivity growth occurs, the cost of operating a machine rises, so older machines are less likely to be utilized.

Following Golosov et al. (2014), the climate block of the model adds a climate damage function that reduces productivity in all sectors. Climate damage is a function of the carbon stock that accumulates based on fossil fuel usage. The damages are external and hence are not taken into account in the optimality conditions that govern production, investment, and household supply of labor and capital. This section provides a complete specification of the technology and optimality conditions for each of the three sectors: final goods, dirty energy and clean energy, along with the specification of the household problem and the climate damage function.

3.1 Energy sector

The energy sector consists of two types of energy producers. A dirty energy producer produces energy E^D using capital, labour and dirty energy and a green energy producer produces energy E^G from renewable sources using only capital and labour.

Dirty and clean energy are assumed to be imperfect substitutes so that the total energy input to the final goods sector is a CES aggregate of the two energy sources:

$$E_t = \left({E_t^D}^{\frac{\epsilon-1}{\epsilon}} + {E_t^G}^{\frac{\epsilon-1}{\epsilon}} \right)^{\frac{\epsilon}{\epsilon-1}}. \tag{1}$$

Let P_t^G and P_t^D denote the price of clean and dirty energy. Input demands satisfy

$$\frac{E_t^k}{E_t} = \left(\frac{P_t^k}{P_t^E} \right)^{\epsilon} \quad for \quad k = G, D.$$

The cost-minimizing energy bundle then implies the energy price:

$$P_t^E = \left(P_t^{G\,1-\epsilon} + P_t^{D\,1-\epsilon}\right)^{\frac{1}{1-\epsilon}} \tag{2}$$

3.2 Dirty energy sector

The dirty energy sector produces energy from vintage-specific machines that combine labour, capital and fuel. Each machine employs one unit of labor at full capacity. For machines built in period $t-j$, the capital-to-fuel ratio k_{t-j}^D and the fuel-to-labour ratio f_{t-j} are chosen upon installation and hence fixed at time $t-j$.

Technological change in the dirty energy sector reflects three forces: disembodied technology and climate damages that affect machines of all vintages equally and vintage-specific technology embodied in machines. Define aggregate disembodied productivity in the dirty energy sector as $A_{d,t} = Z_{d,t} \times D_t$ where $Z_{d,t}$ denotes disembodied technology and D_t denotes damages from accumulated carbon stocks. Upon installation, machines are subject to an idiosyncratic productivity shock $\theta_{i,t-j}^D$ drawn from a log-normal distribution:

$$log(\theta_{i,t-j}^D) \sim \mathcal{N}(log(\theta_{t-j}^D) - \frac{1}{2}\sigma_D^2, \sigma_D^2)$$

The mean of this distribution $\theta_{t-j}^D = E(\theta_{i,t-j}^D)$ denotes the technology embodied in vintage machines $t-j$.

Given the capacity constraint, $L_{i,t,t-j} \leq 1$, a machine i of vintage j at time t produces dirty energy according to the Leontief production function

$$E_{i,t,t-j}^D = X_{i,t,t-j}^D \min[L_{i,t,t-j}^D(X_{i,t,t-j}^D), 1]$$

where machine labour productivity $X_{i,t,t-j}^D$ is a Cobb-Douglas function of technology, climate damages and the capital-to-fuel and fuel-to-labor ratios:

$$X_{i,t,t-j}^D = \left(A_{d,t}\theta_{i,t-j}^D\right)^{1-\alpha^D} (k_{t-j}^D)^{\lambda^D} f_{t-j}^{\alpha^D}.$$

3.2.1 Machine Utilization

Let W_t denote the wage rate and P_t^f the price of fuel. The cost of operating a vintage $t-j$ machine at full capacity is $W_t + P_t^f f_{t-j}$. A machine is used $(L(X_{i,t,t-j}^D) = 1)$ if machine revenue exceeds cost: $P_t^D X_{i,t,t-j}^D > (W_t + P_t^f f_{t-j})$. Define the cutoff value

$$z_{t,t-j}^D = \frac{1}{\sigma_D}\left[log(W_t + P_t^f f_{t-j}) - log(P_t^D X_{t,t-j}^D) + \frac{1}{2}\sigma_D^2\right], \tag{3}$$

the proportion of machines in $t-j$ that are used in time t is

$$Pr[P_t^D X_{i,t,t-j}^D > (W_t + P_t^f f_{t-j})] = 1 - \Phi(z_{t,t-j}^D).$$

Let $X_{t,t-j}^D = E(X_{i,t,t-j}^D)$ denote the unconditional mean of labour productivity in time t for machines built in period $t-j$. Given the log-normal distribution of machine productivity, the expected output of such a machine conditional on utilization is

$$E\left[X_{i,t,t-j}^D | P_t^D X_{i,t,t-j}^D > (W_t + P_t^f f_{t-j})\right] = \frac{(1-\Phi(z_{t,t-j}^D - \sigma_D))}{(1-\Phi(z_{t,t-j}^D))} X_{t,t-j}^D.$$

3.2.2 Optimal Input Choices and Factor Shares

Dirty machines exogenously fail at rate δ^D per period. Expected net income at $t+s$ from a machine that is installed at time t is

$$\pi^D_{t+s,t} = (1-\delta^D)^{s-1}\left\{\left[1-\Phi\left(z^D_{t+s,t}-\sigma_D\right)\right]P^D_{t+s}X^D_{t+s,t} - \left[1-\Phi\left(z^D_{t+s,t}\right)\right]\left(W_{t+s}+P^f_{t+s}f_t\right)\right\}. \tag{4}$$

Define the (ex-dividend) expected market value of a vintage $t-j$ machine at time t as the present value of the expected profit stream $\pi^D_{t+s,t}$, discounted using the household discount factor $m_{t,t+s}$

$$V^D_{t,t-j} = \sum_{s=1}^{\infty} m_{t,t+s}\pi^D_{t+s,t-j}.$$

Machine producers choose k^D_t and f_t to maximize the expected discounted value of profits of a new machine $V^D_{t,t}$. Owing to sector-specific adjustment costs in the production of new investment goods, the price of a new machine, $P^D_{I,t}$, may deviate from unity. The optimality conditions for k^D_t, f_t may be expressed as

$$P^D_{I,t}k^D_t f_t = \sum_{s=1}^{M} m_{t,t+s}(1-\delta^D)^{s-1}\left\{\lambda^D\left[1-\Phi\left(z^D_{t+s,t}-\sigma_D\right)\right]P^D_{t+s}X^D_{t+s,t}\right\} \tag{5}$$

$$P^D_{I,t}k^D_t f_t = \sum_{s=1}^{M} m_{t,t+s}(1-\delta^D)^{s-1}\left\{\alpha^D\left[1-\Phi\left(z^D_{t+s,t}-\sigma_D\right)\right]P^D_{t+s}X^D_{t+s,t} - \left[1-\Phi\left(z^D_{t+s,t}\right)\right]P^f_{t+s}f_t\right\}. \tag{6}$$

Free entry implies the zero profit condition:

$$P^D_{I,t}k^D_t f_t = \sum_{s=1}^{M} m_{t,t+s}(1-\delta^D)^{s-1}\left\{\left[1-\Phi\left(z^D_{t+s,t}-\sigma_D\right)\right]P^D_{t+s}X^D_{t+s,t} - \left[1-\Phi\left(z^D_{t+s,t}\right)\right]\left(W_{t+s}+P^f_{t+s}f_t\right)\right\}. \tag{7}$$

Combining the optimality conditions with the free-entry condition delivers the following labour and fuel share equations:

$$1-\alpha^D = \frac{\sum_{s=1}^{M} m_{t,t+s}(1-\delta^D)^{s-1}\left\{\left[1-\Phi\left(z^D_{t+s,t}\right)\right]W_{t+s}\right\}}{\sum_{s=1}^{M} m_{t,t+s}(1-\delta^D)^{s-1}\left\{\left[1-\Phi\left(z^D_{t+s,t}-\sigma\right)\right]P^D_{t+s}X^D_{t+s,t}\right\}}$$

$$\alpha^D-\lambda^D = \frac{\sum_{s=1}^{M} m_{t,t+s}(1-\delta^D)^{s-1}\left\{\left[1-\Phi\left(z^D_{t+s,t}\right)\right]P^f_{t+s}f_t\right\}}{\sum_{s=1}^{M} m_{t,t+s}(1-\delta)^{s-1}\left\{\left[1-\Phi\left(z^D_{t+s,t}-\sigma\right)\right]P^D_{t+s}X^D_{t+s,t}\right\}}$$

Because of the ex-post fixity of capital-to-energy and fuel-to-labour choices, the labour and fuel input cost shares are determined by the ratio of the expected present value of input costs to revenue.

3.2.3 Input Demands and Energy Output

Let Q^D_{t-j} denote the number of dirty energy machines installed at time $t-j$ so that $(1-\delta^D)^{j-1}Q^D_{t-j}$ is the undepreciated stock of vintage $t-j$ machines at time t. The total input

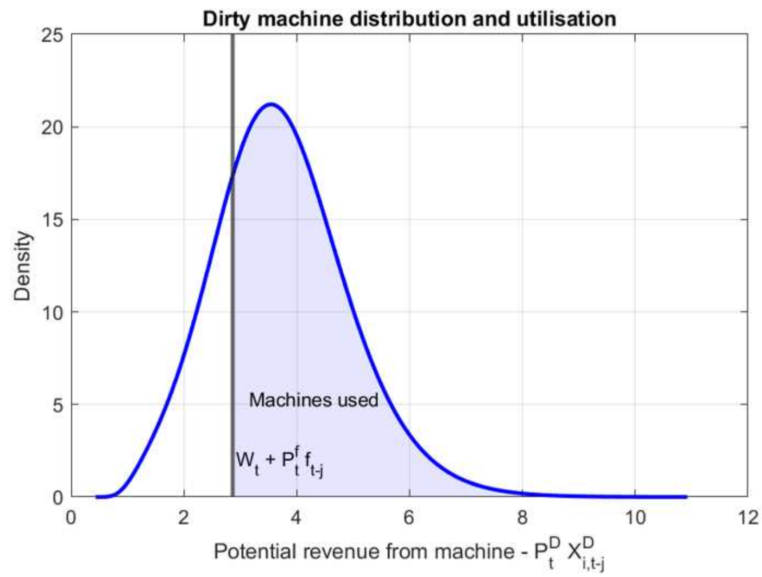


Figure 8: Dirty machine distribution

demand for labour L_t^D and fuel F_t in the dirty energy sector are obtained by summing the utilization-adjusted input requirements across vintages:

$$L_t^D = \sum_{j=1}^{M}\left\{\left[1-\Phi\left(z_{t,t-j}^D\right)\right](1-\delta^D)^{j-1}Q_{t-j}^D\right\} \tag{8}$$

$$F_t = \sum_{j=1}^{M}\left\{\left[1-\Phi\left(z_{t,t-j}^D\right)\right]\times(1-\delta^D)^{j-1}Q_{t-j}^D f_{t-j}\right\}. \tag{9}$$

Similarly, total energy output from the dirty-energy sector satisfies:

$$E_t^D = \sum_{j=1}^{M}\left\{\left[1-\Phi\left(z_{t,t-j}^D-\sigma_D\right)\right]\times(1-\delta^D)^{j-1}Q_{t-j}^D X_{t,t-j}^D\right\}. \tag{10}$$

Figure 8 displays the steady-state distribution of machines for the dirty energy sector along with the steady-state cut off value determined by $W_t + P^F f_t$ based on the calibration chosen below. Machine revenue $P_t^D X$ is displayed along the horizontal axis. The height of the curve is then determined by the number of undepreciated machines that remain in steady-state for a given productivity X. To obtain this curve we need only to track the steady-state values of $\left[Q_{t-j}^D, X_{t-j}^D\right]_{j=0}^{\infty}$. The shaded area to the right of the cut off represents the measure of machines that are utilized. As the economy grows, the wage rises and the utilization rate of less productive machines falls to zero. At the same time, new machines with higher productivity are built, expanding the curve to the right. The second line captures the effect of new investment on the machine distribution.

In summary, given a sequence of prices $\left[w_{t+s}, P_{t+s}^D, P_{t+s}^F\right]_{s=0}^{\infty}$ and technological indices $\left[A_{t+s}^d, \theta_{t+s}^d\right]_{s=0}^{\infty}$, along with the state variables $\left[Q_{t-j}^D, X_{t-j}^D\right]_{j=0}^{\infty}$ that summarize the existing distribution of machines, the dirty energy sector can be fully characterized by equation 3, the static optimal utilization decision, the forward-looking equations 5, 6 and 7 that determine the optimal choices for the capital-to-energy ratio and fuel-to-energy ratio, along with the free entry condition and the backward-looking equations 8, 9 and equation 10 that sum over the existing machine distribution to determine labor and fuel inputs along with dirty energy output.

3.2.4 Vintage Model

An alternative approach to respect the vintage structure of capital is the putty-putty framework formulated by Solow (1959). In this formulation, output per vintage and capital per vintage satisfy

$$\begin{aligned} E^D_{t,t-j} &= A_{d,t}\theta^D_{v,t-j}(K^D_{t,t-j})^{\lambda^D}(F_{t,t-j})^{\alpha^D-\lambda^D}(L^D_{t,t-j})^{1-\alpha^D} \\ K^D_{t,t-j} &= (1-\delta^D)^{t-j-1}I^D_{t-j}. \end{aligned}$$

where I^D_{t-j} are total capital expenditures for vintage $t-j$ and $\theta^{D,v}_{t-j}$ is an index of the technology embodied in the vintage $t-j$ aggregate capital stock. Summing across vintages gives total energy output from the dirty energy sector:

$$E^D_t = \sum_{j=1}^{\infty} E^D_{t,t-j}.$$

Because labor and fuel productivity are equalized across machines both within and across vintages, one can represent the vintage production structure in terms of a capital aggregate that combines with labour, fuel, and technology in a Cobb-Douglas production function. In particular, it is straightforward to show that the vintage model is equivalent to a model with an aggregate capital stock K^D_t subject to investment-specific technological change:

$$K^D_t = (1-\delta^D)K^D_{t-1} + (\theta^{D,v}_{t-1})^{\frac{1}{\alpha}} I^D_t$$

and production function:

$$E^D_t = A^d_t \left(K^D_t\right)^{\lambda^D} (F_t)^{\alpha^D-\lambda^D}(L^D_t)^{1-\alpha^D}.$$

In contrast, in the putty clay model, machine productivities cannot be equalized either within or across vintages. Hence there is no aggregate representation of the capital stock and we therefore need to track the entire distribution of machines through $\left[Q^D_{t-j}, X^D_{t-j}\right]_{j=0}^{\infty}$.

3.3 Green energy sector

The renewable/green energy sector produces green energy E^G_t using only capital and labour, and no fossil fuels, using putty-clay technology. This is the same as the original set-up of Gilchrist and Williams (2000) with the addition of disembodied technological change along with damages induced by carbon accumulation. The formulation thus mirrors the dirty energy sector absent the fossil-fuel decision and use.

Given a capital intensity k^G_{t-j} for vintage $t-j$, a machine i of vintage $t-j$ produces green energy at time t according to the Leontief production function:

$$E^G_{i,t,t-j} = X^G_{i,t,t-j} \min[L^G_{i,t,t-j}(X^G_{i,t,t-j}), 1]$$

where $X^G_{i,t,t-j}$ is labour productivity:

$$X^G_{i,t,t-j} = \left(A_{g.t}\theta^G_{i,t-j}\right)^{1-\alpha^G} (k^G_{t-j})^{\alpha^G},$$

$A_{g,t} = Z_{g,t}D_t$ denotes disembodied total factor productivity inclusive of carbon damages, and $\theta_{i,t-j}$ is an idiosyncratic draw from the sector-specific log-normal distribution

$$log(\theta^G_{i,t-j}) \sim \mathcal{N}(log(\theta^G_{t-j}) - \frac{1}{2}\sigma^2_G, \sigma^2_G).$$

Here $\theta^G_{t-j} = E(\theta^G_{i,t-j})$ indexes the level of technology embodied in green energy machines.

Projects with productivity $P^G_t X^G_{i,t,t-j} \geq W_t$ are used at full capacity, the rest of the machines are left idle. The proportion of machines of vintage $t-j$ in use is therefore given by:

$$Pr(P^G_t X^G_{i,t,t-j} > W_t | W_t, P^G_t) = 1 - \Phi(z^G_{t,t-j})$$

with

$$z^G_{t,t-j} = \frac{1}{\sigma}\left[log(W_t) - log P^G_t X^G_{t,t-j} + \frac{1}{2}\sigma^2\right] \tag{11}$$

Expected profits at time $t+s$ from a machine built in time t are

$$\pi^G_{t+s,t} = (1-\delta^G)^{s-1}\left\{\left[1-\Phi\left(z^G_{t+s,t} - \sigma_G\right)\right] P^G_{t+s}X^G_{t+s,t} - \left[1-\Phi\left(z^G_{t+s,t}\right)\right](W_{t+s})\right\} \tag{12}$$

Define the (ex-dividend) expected market value of a vintage $t-j$ machine at time t as the present value of the expected profit stream $\pi^D_{t+s,t}$, discounted using the household discount factor $m_{t,t+s}$

$$V^G_{t,t-j} = \sum_{s=1}^{\infty} m_{t,t+s}\pi^G_{t+s,t-j}.$$

Machine producers choose k^G_t to maximize $V^G_{t,t}$ resulting in the optimality condition for capital intensity k^G_t

$$P^G_{I,t}k^G_t = \sum_{s=1}^{M} m_{t,t+s}(1-\delta^G)^{s-1}\left\{\lambda^G\left[1-\Phi\left(z^G_{t+s,t} - \sigma_G\right)\right] P^G_{t+s}X^G_{t+s,t}\right\}. \tag{13}$$

This optimality condition along with the zero profit condition

$$P^G_{I,t}k^G_t = \sum_{s=1}^{M} m_{t,t+s}(1-\delta^G)^{s-1}\left\{\left[1-\Phi\left(z^G_{t+s,t} - \sigma_G\right)\right] P^G_{t+s}X^G_{t+s,t} - \left[1-\Phi\left(z^G_{t+s,t}\right)\right](W_{t+s})\right\} \tag{14}$$

deliver the labour share equation:

$$1-\alpha^G = \frac{\sum_{s=1}^{M} m_{t,t+s}(1-\delta^G)^{s-1}\left\{\left[1-\Phi\left(z^G_{t+s,t}\right)\right] W_{t+s}\right\}}{\sum_{s=1}^{M} m_{t,t+s}(1-\delta^G)^{s-1}\left\{\left[1-\Phi\left(z^G_{t+s,t} - \sigma\right)\right] P^G_{t+s}X^G_{t+s,t}\right\}}.$$

Finally the total labour requirement and green energy output are:

$$L_t^G = \sum_{j=1}^{M} \left\{ \left[1 - \Phi \left(z_{t,t-j}^G \right) \right] (1-\delta^G)^{j-1} Q_{t-j}^G \right\} \tag{15}$$

$$E_t^G = \sum_{j=1}^{M} \left\{ \left[1 - \Phi \left(z_{t,t-j}^G - \sigma^G \right) \right] \times (1-\delta^G)^{j-1} Q_{t-j}^G X_{t,t-j}^G \right\} \tag{16}$$

where Q_{t-j}^G is the quantity of green energy machines of vintage $t-j$.

3.4 Final goods producers

The final goods producers combine capital, labour, and energy to produce output that is either consumed by households or used to produce new machines in the final goods or energy sectors of the economy. The putty-clay version of the final goods problem follows closely the set-up of the dirty energy firm. Let k_{t-j}^{fg} and e_{t-j}^{fg} denote the capital-to-energy and energy-to-labour intensities of machines of vintage $t-j$.

A machine i of vintage $t-j$ produces final goods at time t

$$Y_{i,t,t-j}^{fg} = X_{i,t,t-j}^{fg} \min[L_{i,t,t-j}^{fg}(X_{i,t,t-j}^{fg}), 1]$$

where machine labour productivity $X_{i,t,t-j}^{fg}$ is

$$X_{i,t,t-j}^{fg} = \left(A_{fg,t} \theta_{i,t-j}^{fg} \right)^{1-\alpha^{fg}} (k_{t-j}^{fg})^{\lambda^{fg}} e_{t-j}^{\alpha^{fg}}.$$

with

$$log(\theta_{i,t-j}^G) \sim \mathcal{N}(log(\theta_{t-j}^G) - \frac{1}{2}\sigma_G^2, \sigma_G^2)$$

Again, the mean of this distribution $\theta_{t-j}^G = E(\theta_{i,t-j}^G)$ denotes the technology embodied in final goods machines of vintage $t-j$.

The cost of operating a vintage $t-j$ machine at full capacity is $W_t + P_t^e e_{t-j}$ Define the cutoff value

$$z_{t,t-j}^{fg} = \frac{1}{\sigma_{fg}} \left[log(W_t + P_t^e e_{t-j}) - log(P_t^{fg} X_{t,t-j}^{fg}) + \frac{1}{2}\sigma_{fg}^2 \right], \tag{17}$$

the proportion of machines in $t-j$ that are used in time t is

$$Pr[P_t^{fg} X_{i,t,t-j}^{fg} > (W_t + P_t^e e_{t-j})] = 1 - \Phi(z_{t,t-j}^{fg}).$$

Let $X_{t,t-j}^{fg} = E(X_{i,t,t-j}^{fg})$ denote the unconditional mean of labour productivity in time t for machines built in period $t-j$. The expected output of such a machine conditional on utilization is

$$E\left[X_{i,t,t-j}^{fg} | P_t^{fg} X_{i,t,t-j}^{fg} > (W_t + P_t^e e_{t-j}) \right] = \frac{(1 - \Phi(z_{t,t-j}^{fg} - \sigma_{fg}))}{(1 - \Phi(z_{t,t-j}^{fg}))} X_{t,t-j}^{fg}.$$

Given the failure rate δ^{fg} of final goods machines, expected net income at $t+s$ from a machine that is installed at time t is

$$\pi^{fg}_{t+s,t} = (1-\delta^{fg})^{s-1}\left\{\left[1-\Phi\left(z^{fg}_{t+s,t}-\sigma_{fg}\right)\right]P^{fg}_{t+s}X^{fg}_{t+s,t} - \left[1-\Phi\left(z^{fg}_{t+s,t}\right)\right]\left(W_{t+s}+P^{e}_{t+s}e_t\right)\right\} \tag{18}$$

Define the (ex-dividend) expected market value of a vintage $t-j$ machine at time t as the present value of the expected profit stream $\pi^{D}_{t+s,t}$, discounted using the household discount factor $m_{t,t+s}$

$$V^{fg}_{t,t-j} = \sum_{s=1}^{\infty} m_{t,t+s}\pi^{fg}_{t+s,t-j}.$$

Machine producers choose k^{fg}_t and e_t to maximize the expected discounted value of profits of a new machine $V^{fg}_{t,t}$. The optimality conditions for the input ratios k^{fg}_t, e_t may be expressed as

$$P^{fg}_{I,t}k^{fg}_t e_t = \sum_{s=1}^{M} m_{t,t+s}(1-\delta^{fg})^{s-1}\left\{\lambda^{fg}\left[1-\Phi\left(z^{fg}_{t+s,t}-\sigma_{fg}\right)\right]P^{fg}_{t+s}X^{fg}_{t+s,t}\right\} \tag{19}$$

$$P^{fg}_{I,t}k^{fg}_t e_t = \sum_{s=1}^{M} m_{t,t+s}(1-\delta^{fg})^{s-1}\left\{\alpha^{fg}\left[1-\Phi\left(z^{fg}_{t+s,t}-\sigma_{fg}\right)\right]P^{fg}_{t+s}X^{fg}_{t+s,t} - \left[1-\Phi\left(z^{fg}_{t+s,t}\right)\right]P^{e}_{t+s}e_t\right\}. \tag{20}$$

Free entry implies the zero profit condition:

$$P^{fg}_{I,t}k^{fg}_t e_t = \sum_{s=1}^{M} m_{t,t+s}(1-\delta^f g)^{s-1}\left\{\left[1-\Phi\left(z^{fg}_{t+s,t}-\sigma_{fg}\right)\right]P^{fg}_{t+s}X^{fg}_{t+s,t} - \left[1-\Phi\left(z^{fg}_{t+s,t}\right)\right]\left(W_{t+s}+P^{e}_{t+s}e_t\right)\right\}. \tag{21}$$

We can again combine the optimality conditions for k^{fg}_t and e_t with the free entry condition to obtain expressions for optimal input shares.

Let Q^{fg}_{t-j} denote the number of final goods machines installed at time $t-j$ so that $(1-\delta^{fg})^{j-1}Q^{fg}_{t-j}$ is the undepreciated stock of vintage $t-j$ machines at time t. The total input demand for labour L^{fg}_t and energy E_t in the final goods sector are :

$$L^{fg}_t = \sum_{j=1}^{M}\left\{\left[1-\Phi\left(z^{fg}_{t,t-j}\right)\right](1-\delta^{fg})^{j-1}Q^{fg}_{t-j}\right\} \tag{22}$$

$$E_t = \sum_{j=1}^{M}\left\{\left[1-\Phi\left(z^{fg}_{t,t-j}\right)\right]\times(1-\delta^{fg})^{j-1}Q^{fg}_{t-j}e_{t-j}\right\}. \tag{23}$$

while total final goods output satisfies:

$$Y^{fg}_t = \sum_{j=1}^{M}\left\{\left[1-\Phi\left(z^{fg}_{t,t-j}-\sigma_{fg}\right)\right]\times(1-\delta^{fg})^{j-1}Q^{fg}_{t-j}X^{fg}_{t,t-j}\right\}. \tag{24}$$

3.5 Climate Damage Function

In order to assess the welfare implications of imposing taxes and subsidies to reduce fossil fuel usage, we introduce a climate block into the model which models the endogenous responses of damages from climate change. We model the externality from climate change as reducing to the productivity, following the approach of Nordhaus's DICE models. We adopt the tractable approach to modelling the climate block of IAMs proposed by Golosov et al. (2014). The climate damages are a function of the stock of carbon S_t that has been produced previously by the fossil fuel usage of the dirty energy sector, subject to a reduced form representation of how this stock is partly absorbed and depleted by the environment.

We make some minor modifications to the framework of Golosov et al. (2014). Firstly, we substitute their functional form for damages $\exp\{-\gamma(S_t - \bar{S})\}$ with the damage function :

$$D_t = \frac{S_t}{\tilde{S}}^{-\zeta} \tag{25}$$

Where ζ and $\tilde{S}$ are the damage function parameter and a normalisation of the carbon stock. We chose these parameters to closely match the damage function specifications of the RICE (2016) and Golosov et al. (2014) damage function calibrations. This functional form is able to closely match damages incorporated in their models over ranges of the carbon stock reached in our model simulations. Note that this is specified as a function of the industrial emissions, i.e. as deviations from the pre-industrial atmopsheric carbon stock and any non-industrial carbon emissions. This functional form is more compatible with a balanced growth path. We allow all sectors, not only the final sector, to be subject to climate damages, with labour productivity reduced by A_t^{fg}, A_t^d and A_t^g being multiplied by D_t.

The carbon stock S_t accumulates based on past fossil fuel use:

$$S_t = \sum_{s=0}^{t+T} (1 - d_s) F_{t-s} \tag{26}$$

Here $(1 - d_s)$ reflects the depreciation of previous carbon stocks. We make a minor modification to the depreciation in Golosov et al. (2014), allowing the long-term presence of carbon in the atmosphere to be extremely persistent (over centuries), but not completely permanent, as in their specification. This allows the existence of a steady state in the model where fossil fuel usage is non-zero, but in our calibration still closely matches their depreciation dynamics.

$$1 - d_s = (1 - \varphi_L)\varphi_0(1 - \varphi_1)^s + \varphi_L(1 - \varphi_2)^s$$

where φ_L: is the share of carbon emitted that remains in the atmosphere extremely persistently, $(1 - \varphi_2)^s$ controls this persistence and is our modification to the earlier framework. $1 - \varphi_0$ exits the atmosphere immediately, and the remaining share decays geometrically.

3.6 Households

There is a representative household who owns the capital, obtains utility from consumption and disutility from supplying labour. The household can buy and sell sector-specific claims on the portfolio of capital that pay out π_t^x in dividends each period for $x = D, G, fg$ where π_t^x are total profits paid out across vintages in sector x at time t

$$\pi_t^x = \sum_{j=1}^{M} \pi_{t,t-j}^x.$$

Let s_t^x define the number of shares on the portfolio of claims to sector x held by the household, and V_t^x the ex-dividend market value of this portfolio:

$$V_t^x = \sum_{j=1}^{M} V_{t,t-j}^x$$

Households choose a sequence of consumption, labour and portfolio shares to maximize

$$\sum_{t=0}^{\infty} \left\{ \beta^t \frac{1}{1-\eta} \times \left[C_t \left(1 - L_t\right)^{\varphi} \right]^{1-\eta} \right\}$$

subject to

$$C_t + \sum_{x=D,G,fg} s_{t+1}^x V_{t+1}^x = W_t L_t + \sum_{x=D,G,fg} s_t^x \left(\pi_t^x + \left(1 - \delta^x\right) V_t^x \right)$$

The household optimality conditions may be expressed as

$$W_t = \varphi \frac{C_t}{1 - L_t} \tag{27}$$

$$m_{t,t+1} = \frac{1}{1 + r_{t+1}} = \beta \frac{U_1(C_{t+1}, L_{t+1})}{U_1(C_t, L_t)} \tag{28}$$

with rates of return on assets equalized across sectors:

$$1 + r_{t+1} = \frac{(1 - \delta^x) V_{t+1}^x + \pi_{t+1}^x}{V_t^x} \quad for \;\; x = D, G, fg. \tag{29}$$

3.7 Investment, fuel costs and the aggregate resource constraint

Absent adjustment costs, immediate changes in sector-specific technology or taxes imply implausibly large investment and disinvestment patterns across sectors. To address this concern, the model allows for adjustment costs when investment rises above past investment. In particular, increasing investment in sector x relative to last period entails a per-unit of investment cost:

$$C_t^x = \min \left(\frac{\phi_I}{2} \left(\frac{I_t^x}{(1 + g_{x,I} + \delta_x) I_{t-1}^x} \right) - 1, 0 \right)$$

where $g_{x,I}$ is the steady-state growth rate of investment in sector x and sectoral investment reflects both the number of machines and the quality of machines that are constructed:

$$\begin{aligned} I_t^D &= Q_t^D k_t^D f_t \quad \geq 0 \\ I_t^G &= Q_t^G k_t^G \qquad \geq 0 \\ I_t^{fg} &= Q_t^{fg} k_t^{fg} e_t \quad \geq 0. \end{aligned}$$

This specification implies that the marginal cost of investment is

$$P_t^{x,I} = 1 + \min\left(\phi_I\left(\frac{I_t^x}{(1+g_{x,I}+\delta_x)I_{t-1}^x}\right), 0\right)$$

so that there are zero adjustment costs at the steady-state that rise when investment exceeds its steady-state level. The model also imposes a lower bound of zero on investment so that the investment in past machines is irreversible. The asymmetric nature of the adjustment cost is then consistent with the notion that capacity constraints on the production of new investment goods bind during rapid expansions.

Fossil fuels are assumed to be abundant but costly to extract. In particular, a unit of fuel can be extracted at a constant cost P^F in terms of current units of final goods output. These assumptions imply that output in the final goods sector is used for consumption, investment inclusive of adjustment costs, and fossil fuel extraction:

$$Y_t = C_t + \sum_{x=fg,D,G} (1 + C_t^x)\, I_t^x + P^f F_t \tag{30}$$

3.8 Calibration

Table 1 displays the model parameters chosen for the calibration. The model parameters contain macro parameters that govern household behavior along with parameters that determine the importance of both fuel and energy for the aggregate economy, and the relevance of the putty-clay mechanism. Household parameters include the discount factor β, the intertemporal elasticity of substitution η, and the inverse-Frisch labor supply elasticity. These are chosen to be 0.9925, 2/3, and 3, respectively, which are standard values in the literature. We assume that growth in the final goods sector is equally split between growth in disembodied technology and technology embodied in new capital goods, and we calibrate the trend growth rate of final output to two percent. Maximum utilization rates in the final goods sector are on the order of 0.88 which implies setting $\sigma_{fg} = 0.33$.

The depreciation schedule along with the rate of growth of embodied technology and the dispersion in idiosyncratic project outcomes all influence the extent to which the model displays putty-clay features. The depreciation rate in both the final goods and green energy sector is set to 10 percent per year. The depreciation rate in the dirty energy sector is set to 5 percent to reflect the fact that fossil-fuel plants are longer lived than other forms of capital.

We calibrate σ_d, σ_g and the growth rates of embodied technology in the energy sectors (these growth rates are constrained to be greater than zero and, at most, equal to the growth rate

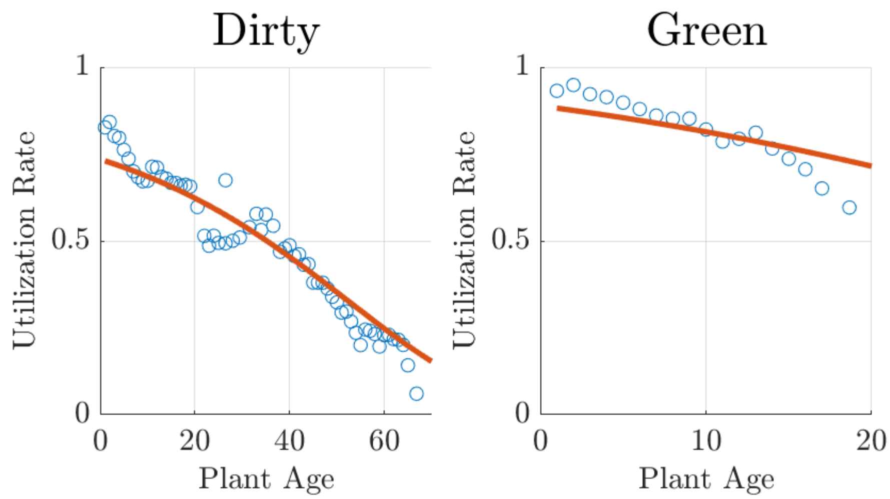


Figure 9: Model fit to age-utilization microdata

of final output) to match micro evidence on the utilization-age schedule of plants in the US energy sector. Figure 9 plots the fit of the model-implied age-utilization schedule to the microdata.

The final goods sector has a labor share $1 - \alpha^{FG} = 0.575$, a capital share $\lambda^{FG} = 0.35$ and an energy share $1 - \alpha^{FG} - \lambda^{FG} = 0.075$. This energy share is consistent with BEA estimates reported for the mid-2000s. Labour share estimates derived from the utility sector suggest setting the labour share for dirty energy $1 - \alpha_d = 0.15$. To match a fuel share of GDP of 0.035, we then choose $\lambda^D = 0.225$.[2] We calibrate the capital share in the green energy sector such that the labor share of (non fuel) cost is the same as in the dirty energy sector, which gives $\alpha_g = 0.6$. Consistent with data from the mid-2000's, the relative productivity of the dirty versus green energy sector is chosen so that green energy accounts for 20 percent of total energy output.

For the climate block, we aim to closely match previous literature; in particular the calibration of the tractable climate block proposed by Golosov et al. (2014). This is because the climate block is not the primary driver of the medium-run economic dynamics that are our key focus in this paper, and so we aim not to innovate here, but show the differences our framework which are comparable to previous IAM calibrations. Given the modifications to the damage function and carbon depreciation schedule discussed in Section 3.5, rather than taking their parameterisation off the shelf, we choose parameters which closely match their calibration. In particular, we choose the damage parameter ζ to match the damage modelled in the DICE calibration matched by Golosov et al. (2014). We also choose the carbon depreciation parameters to match the carbon stock dynamics following a 100 GtC shock to the carbon stock.[3]

[2]The calibration implies a fuel share of non-capital costs of 76%, consistent with EIA data (see https://www.eia.gov/electricity/annual/html/epa_08_04.html).

[3]In particular, we minimise the sum of squared residuals of damages that result from carbon stocks ranging from 700-1500 GtC, between our model and the DICE model. For carbon depreciation, we similarly choose the depreciation parameters to match the carbon stock paths over 500 years following a 100GtC shock

Parameter	Value	Description
$1-\alpha^{FG}$	0.575	Labour share in final goods sector
λ^{FG}	0.35	Capital share in final goods sector
$\alpha^{FG}-\lambda^{FG}$	0.075	Energy income share in final goods sector
$1-\alpha^{D}$	0.15	Labour share in dirty energy sector
λ^{D}	0.225	Capital share in dirty energy sector
$\alpha^{D}-\lambda^{D}$	0.625	Fossil fuel share in dirty sector
α^{G}	0.6	Capital share in green sector
ϵ	2	Elasticity of substitution between energy types
$\delta^{FG}=\delta^{G}$	0.1	Depreciation in final goods and green sectors
δ^{D}	0.05	Depreciation in dirty sector
σ_{FG}	0.33	St dev. log productivity in final goods sector
σ_{D}	0.24	St dev. log productivity in dirty sector
σ_{G}	0.61	St dev. log productivity in green sector
ζ	0.0394	Climate damage in production function (to match DICE 2016)
S^{init}	802	Initial carbon stock
$\tilde{S}$	581	Damage function carbon normalization
φ^{L}	0.3145	Carbon stock depreciation parameter
φ^{0}	0.2917	Carbon stock depreciation parameter
φ^{1}	2.8e-09	Carbon stock depreciation parameter
φ^{2}	0.0023	Carbon stock depreciation parameter
β	0.9925	Discount rate
$1/\eta$	2/3	Elasticity of intertemporal substitution (EIS)
φ	3	Leisure preferences
$g_{\theta d}=g_{\theta g}$	2%	Embodied growth in energy sectors
g_{zfg}	1%	Disembodied growth in final goods sector
g_{y}	2%	GDP growth

Notes: Parameter calibration for the model described in the text.

Table 1: Model Calibration

4 Model Experiments

In this section we consider the effects of technological change in the green energy sector along with tax policies that seek to reduce fossil-fuel use or increase the production of green energy. In all cases, revenues obtained from such taxes are rebated lump-sum to households.

4.1 Green Energy Transition

The first experiment considers the effect of technological change in the green energy sector. In particular, we assume that growth in technology embodied in the green energy sector rises persistently over a thirty-year period. This rise in the growth rate is chosen to match the acceleration in growth exhibited in Figure 7b in the post-2008 period. Although Figure 7b applies directly to solar energy, similar rates of technological change in wind combined with battery storage are leading to rapid gains in technology embodied in clean energy plants. This experiment is therefore informative as to the likely effects of these combined sources of technological change in the green energy sector.

Figure 10 displays responses for both the putty-clay model (blue) and the vintage model (red). The long-run cumulative effect of this persistent increase in the growth rate of green technology is to cause an increase in green energy machine efficiency of XX. Panel A of Figure 10 displays the time path of aggregate variables. The increase in machine productivity

to the carbon stock, by minimising the SSR between our model and that of Golosov et al. (2014).

leads to an expansion in aggregate output, consumption, and investment of approximately eight percent after forty years. The long-run effect on aggregate output reflects two features of the calibration. First, the overall energy share, at 7.5 percent, is relatively small. Second, the initial share of clean energy in energy production is only 20 percent. Because the putty-clay and vintage models share the same steady-state, the long-run effects of an increase in technology are equivalent.

The effects on the energy sector are substantially larger – energy production rises by 180 percent, and since the long-run energy share of output is constant, this implies a decline in energy prices that are equal in magnitude. This long-run transition also implies a shift in production away from dirty energy towards clean energy. Fossil fuel usage declines 30 percent and the carbon stock declines 15 percent over a forty-year horizon. We emphasise that both of these are relative to trend; in absolute terms, the carbon stock continues to rise, but at a slower pace. Although these reductions in fossil-fuel use and carbon stocks are sizeable, this experiment suggests that an increase in the rate of technological change in clean energy as seen in recent experience is unlikely to bring the economy close to net-zero fossil emissions over the next forty years.

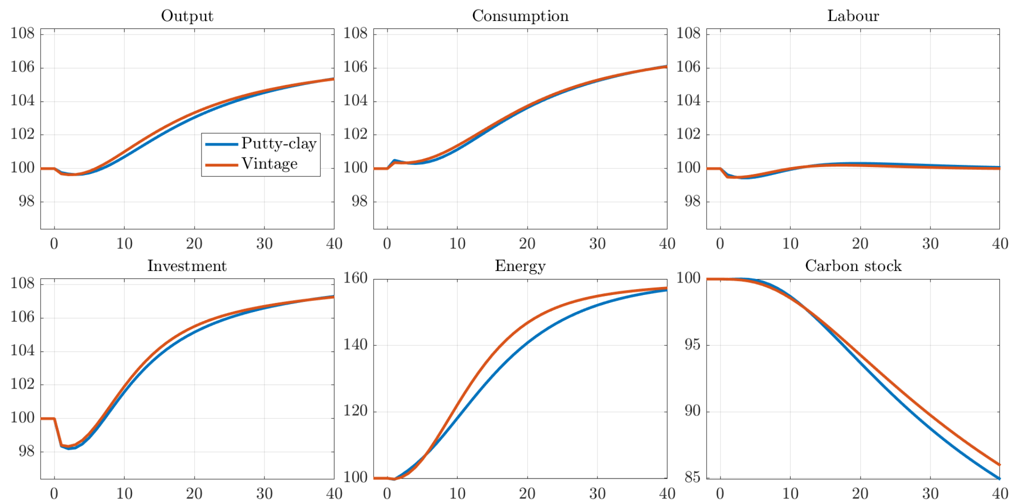
Additional details of the reallocation from dirty to clean energy are shown in panel B of Figure 10. In the long-run, dirty energy production falls by thirty percent while clean energy production increases by more than 400 percent (starting from an initial low level of 20 percent of total energy). Relatedly, clean energy prices fall by 60 percent while dirty energy prices remain essentially unchanged. [4]

Short-run dynamics differ substantially from the long-run. Because productivity gains occur in the future, there is a reduction in investment and output in the short-run along with a modest rise in consumption. Differences between the putty-clay and vintage model are also relatively minor due to the fact that the transition is gradual. The most distinct difference between the two models is a sharper reduction in dirty energy investment and a smaller expansion in clean energy investment along the adjustment path for the putty-clay version. These differences are most pronounced at the medium run horizon of ten to twenty years. This investment pattern also implies less energy production at these horizons.

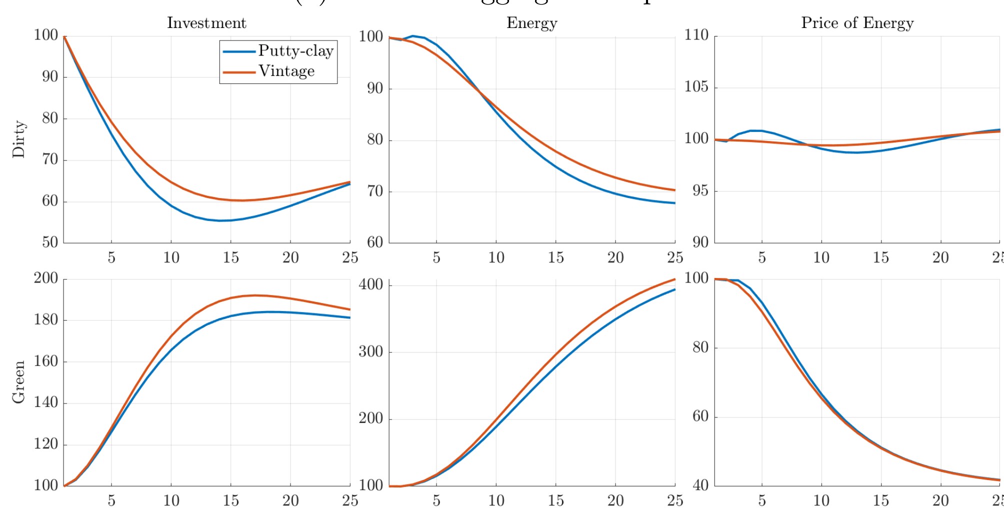
Panel C of Figure 10 shows the time path of variables that are specific to the putty-clay framework. Utilization rates in the green energy sector fall substantially as new machines replace old machines. Utilization rates rise temporarily in the dirty energy sector to compensate for the reduction in capacity due to the delayed investment in clean energy. The putty-clay model also displays the phenomenon of capital widening vs capital deepening. As exogenous machine productivity improves, the economy expands by increasing the quantity of clean machines but reducing their quality as measured by the capital-labor ratio. Over time, this process is reversed and the capital-labour ratio of new machines returns to its long-run value.

In summary, a gradual but persistent improvement in green technology has sizeable effects on energy production and leads to a sizeable reduction in fossil fuel usage. The effects on

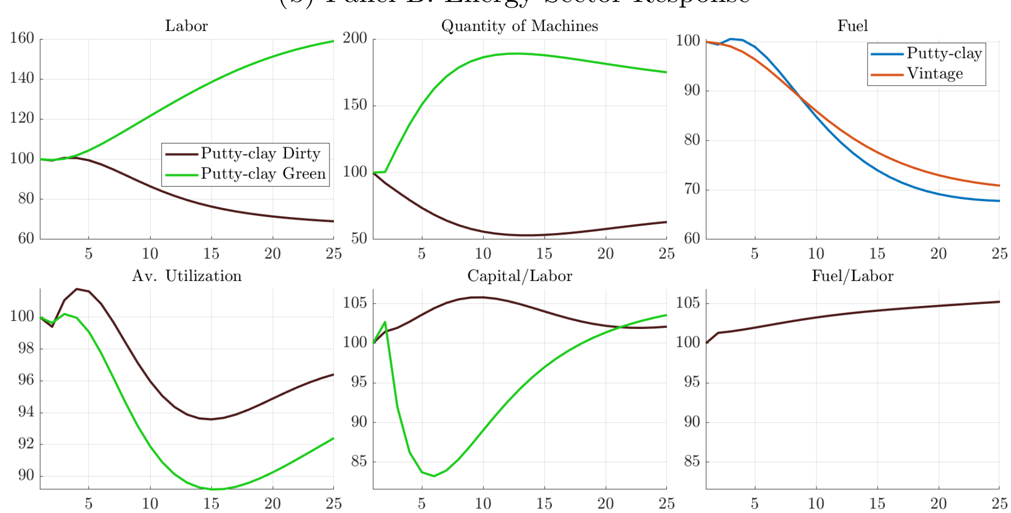
[4] The modest increase of 1 percent in dirty energy prices displayed in panel B of Figure 10 reflects the imperfect substitutability between the two energy types.



(a) Panel A: Aggregate Responses



(b) Panel B: Energy Sector Response



(c) Panel C: Sectoral Decomposition: Putty-Clay Model

Figure 10: Technological Change in the Green Energy Sector

the aggregate economy are modest due to the size of the energy sector. Most notably, one would require much large gains in clean energy productivity for fossil fuel usage to become inconsequential.

4.2 Carbon Tax

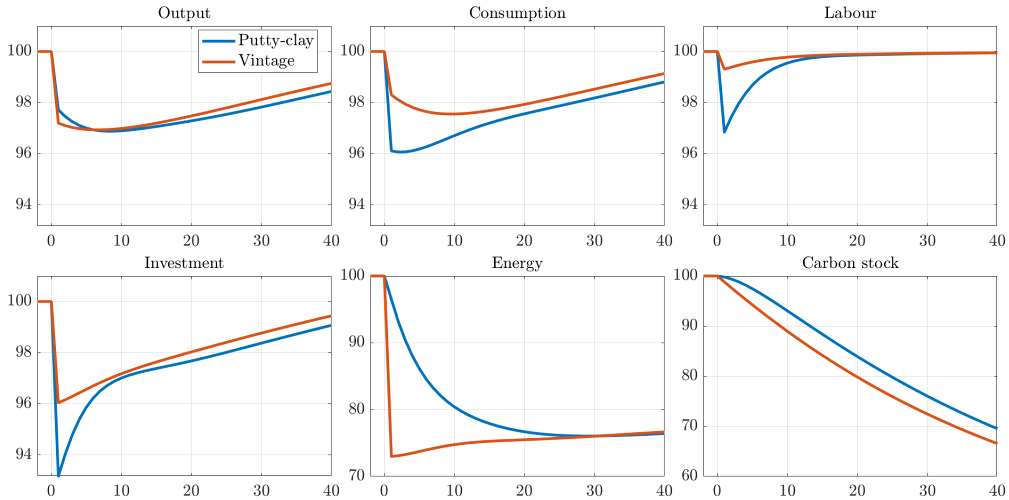
The second experiment considers the effect of an unanticipated carbon tax that permanently doubles the price of fossil fuel. This is equivalent to a carbon tax of approximately USD $75 per ton of carbon emissions, well within the range considered in current policy discussions.

Panel A of Figure 11 displays the aggregate responses to this carbon tax. In the short-run, the tax causes a substantial contraction in economic activity. In the putty-clay model, output falls three percent, investment falls seven percent, and labour falls three percent. In the putty-clay model, there is very little short-run substitutability between energy and other inputs. As a result, total energy only falls gradually in the short-run. As the economy adjusts, energy production continues to fall while output, consumption and investment recover. Labour remains permanently slightly below its initial steady-state however.

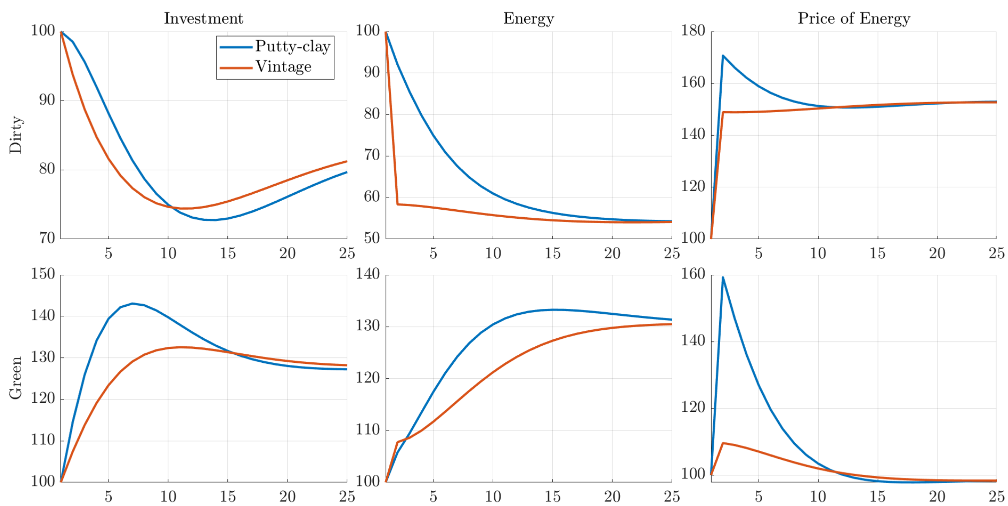
The lack of substitutability between fossil fuels and other inputs also implies sharp rises in energy prices in both the dirty and green energy sectors, along with a gradual decline in dirty energy investment and a shift towards green energy investment. The primary margin of adjustment here is that new machines that are built in the dirty-energy sector reflect the desired long-run fuel-to-labour ratio which falls 50 percent due to the doubling of the fossil fuel price. Thus there are two forces at work to reduce fossil fuel usage: a switch in production towards clean rather than dirty energy and new investments in the dirty-energy sector that embody lower fossil fuel requirements.[5] Fossil fuel usage gradually declines as these transitions occur.

In contrast to the putty-clay model, the fossil fuel usage and energy production exhibit large immediate declines in the vintage model. The forty percent initial decline in energy production is only a few percent greater than the long-run decline. Similarly, the initial decline in fossil fuel usage is only a few percent above the eventual decline of sixty percent. This adjustment occurs with less disinvestment in the dirty sector and less overall investment in the green sector in comparison to the putty-clay model. Because fossil fuel usage and energy production only adjust gradually in the putty-clay framework, the tax is much more effective in reducing the carbon stock in the vintage model relative to the putty-clay model. As discussed in the following section, achieving the same carbon stock reduction over a 25-year horizon requires a 50% larger tax increase in the putty-clay model relative to the vintage model.

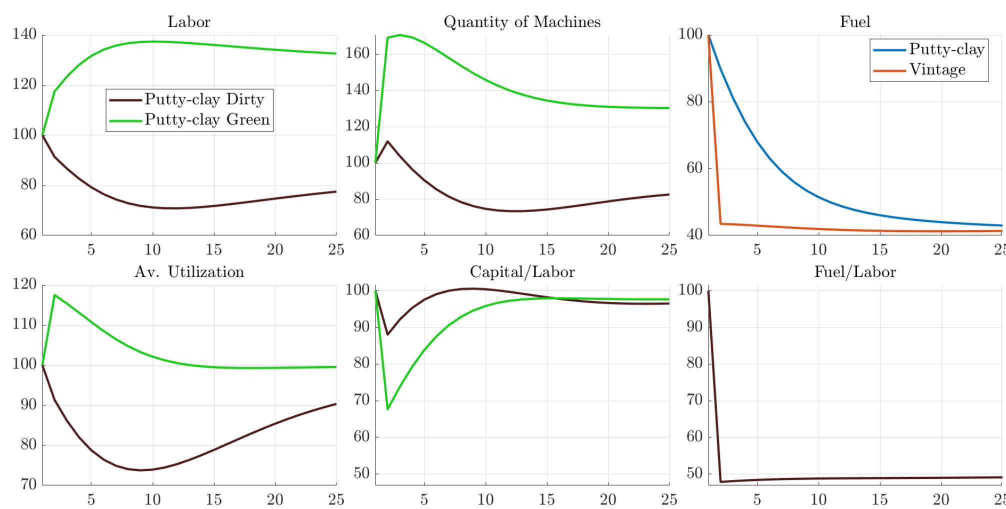
[5]This dynamic of investment shifting overall energy sector capital to be more carbon efficient in both the carbon pricing and green technology improvement cases is consistent with our desciptive evidence and other evidence, such as Bolton and Kacperczyk (2023) that newer capital vintages are more energy efficient.



(a) Panel A: Aggregate Responses



(b) Panel B: Energy Sector Response



(c) Panel C: Sectoral Decomposition: Putty-Clay Model

Figure 11: Carbon Tax

4.3 Stranded assets

In this section, we study stock market responses to a carbon tax. A salient concern in the policy debate on carbon tax implementation (Carney (2015), von Dulong et al. (2023)) is the "stranding" of assets, untapped fossil fuel reserves, and fuel-producing and -burning capital that would be rendered worthless (or at best much less valuable) by taxation. The importance of stranded assets is often motivated by political economy and distributional concerns or by implications for financial stability of large falls in asset values. The concept of a stranded asset is not particularly concrete in a putty-putty world, as we illustrate below. However, with putty-clay technology, we can study the implications of this phenomenon.

We start by defining the value of capital installed prior to the policy change. This is the time t value of the sector I capital stock installed before time t_1, given by the aggregate present discount value of the profits accruing to surviving capital vintages older than t_1:

$$V^I_{t_1,t} = \sum_{j=t-t_1}^{t-t_1+M-1} \sum_{s=1}^{\infty} m_{t,t+s} \pi^I_{t+s,t-j} \left(1-\delta_I\right)^j Q_{I,t-j},$$

where $I \in \{D, G, fg\}$, and profits in the dirty, green and final goods sectors are given respectively by Equations 4, 12, and 18.

Figure 12 shows the transition path of the value of the capital stocks in each of the three sectors in response to a fuel tax, in log deviation from the balanced growth path trajectory. The solid blue and dashed red lines plot the responses to the 100% fuel tax studied in Section 4.2 in the putty-clay and vintage models, respectively.

Valuation of the stranded assets in the dirty sector is clear in the putty-clay model. The value of the dirty sector responds modestly on impact, as the effect of the doubling of fuel prices on profits is initially offset by a jump in the dirty energy price and a fall in the wage. However, over time, dirty assets are stranded in the sense that their value falls persistently, reaching a trough of -20% seven years after the initial shock. This occurs because old, dirty capital is more heavily dependent on heavily taxed fossil fuels. The decline towards lower stock prices is consistent, qualitatively, with findings in the climate finance literature (e.g. Engle et al. (2020), Bolton and Kacperczyk (2021) and Berg et al. (2021),Hsu et al. (2023)) on the response of stock prices to climate policy news and the resulting equilibrium of lower stock prices in high emitting industries. That said, this theoretical approach - and nonsize-dependent, representative firm assumption - can't explain Bolton and Kacperczyk (2021) finding that the absolute level of emissions is more important than emissions intensity. As the energy price rise moderates and wages increase, as more fuel-efficient capital is installed, these older less efficient machines become relatively unprofitable. The value of the green sector jumps on impact due to the large increases in energy prices, while their only variable input cost - labour - becomes cheaper. Valuation changes in the final goods sector are modest. This is again somewhat consistent with evidence from Bolton and Kacperczyk (2021). However, the perfect information transmission means this is because the pass-through of higher fossil fuel costs is small; not because investors may only be paying attention to scope 1 emissions or particular industries, in a manner perhaps more consistent with imperfect market understanding of carbon risk exposure.

In the vintage model the valuation of old capital is relatively unaffected. This is because assets are not really "stranded"; although their older technology becomes more out of date, the fact at input ratios are not fixed means that the dirty energy sector is able to use old capital in a more fuel efficient way. As a result, energy sector valuations only move slightly and mirror each other. There is a small and transitory fall in value in the dirty sector, and the opposite in the green. The value of the final goods sector also increases modestly, because the negative effect of higher energy prices on profits is more than offset by a fall in wages.

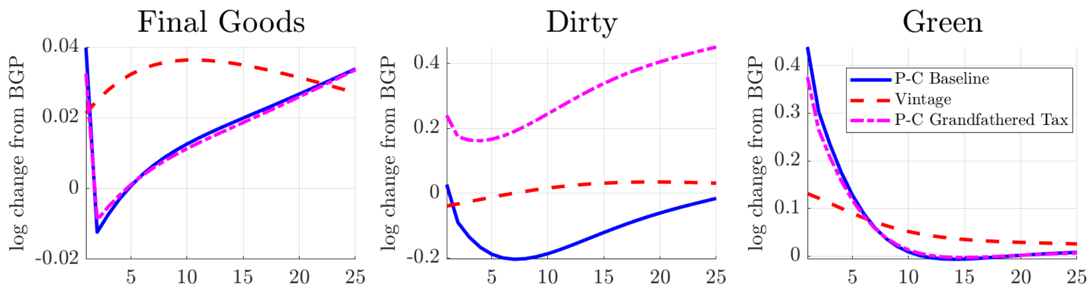


Figure 12: Stock market response to fossil fuel tax

To make the carbon tax more palatable by avoiding these negative effects on valuations, one option is grandfathering; allowing capital installed prior to policy implementation to have reduced or no carbon tax liability. The dashed-dotted magenta lines in Figure 12 plots the responses of values in the putty-clay model, but using the alternative tax policy in which vintages of capital installed before the tax is introduced pay a 20% lower carbon tax rate. This results in the value of old vintages in the dirty sector now increasing on impact, so this alternative policy does not result in stranding of assets. Values in the green and final goods sector respond similarly to the uniform carbon tax case (with more muted responses in both cases). This suggests that a carbon tax policy that partially exempts existing dirty assets from taxation might be preferable from both welfare perspectives and may be more politically feasible.

4.4 Welfare and policy implications

Next, we assess the efficacy and welfare impact of fossil fuel tax policies. For this exercise, we look at the response after 25 years after policy implementation, around the focal point of many international net-zero targets and after most of the short to medium term economic dynamics of the policy implementation have occurred. The left panel of Figure 13 shows the change in carbon stock and welfare impact at 25 years after policy implementation, for a range of fuel tax sizes. Our policy simulation, \$75, or a 100% increase in the price of fossil fuels, is at the centre of the scenarios. Consistent with the baseline results, under a putty-clay model, there is less of a reduction in the carbon stock than under standard, vintage capital framework. We emphasise that these results are deviations from a balanced growth path – the carbon stock is rising, but at a slower rate as a result of the policy than would otherwise be the case. The difference of carbon stocks is material; using these detrended carbon stock path results as the atmospheric carbon stock within the CDICE calibration proposed by Folini et al. (2024), this implies a 0.15°C higher temperature under the putty-clay model, after a 100% fuel tax is imposed.

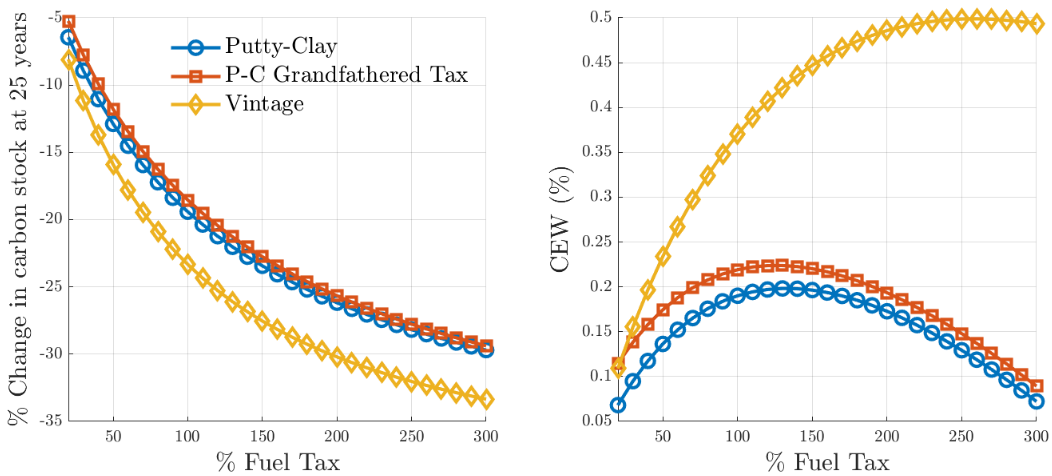


Figure 13: Carbon stocks and consumption equivalent welfare for different fossil fuel tax

The difference in welfare across the two models is substantial. Under the more flexible, vintage model, the carbon tax improves welfare as the externality from climate change is reduced. In the putty-clay model, the welfare improvement from carbon taxation is significantly smaller for all values of the tax, and the optimal taxis much smaller. This is because there is a substantial portion of the pre-existing dirty energy capital stock which is now not profitable to use. This represents a large welfare loss from stranded assets which are mothballed in the aftermath of the policy implementation.

To further illustrate the link between the stranding of assets in the putty-clay model and the welfare implications of a carbon tax, Figure 13 also plots (red squares) the response of the carbon stock and welfare under an alternative policy in which the preinstalled carbon stock is taxed at a 20% discount to the carbon tax levied on fuel use by machines installed after the policy is introduced ("grandfathered tax"). The carbon stock in this alternative scenario falls marginally less than in the baseline taxation, but welfare is higher. Grandfathering the preexisting capital stock improves welfare because it eliminates (or at least reduces) the stranded asset problem, which translates into a smaller fall in consumption on policy implementation. Under our baseline calibration, this short-term improvement offsets the long-term welfare cost of the higher carbon stock.

We then conduct an exercise to see how much larger fossil fuel taxes may need to be to achieve the same carbon stock goals as under the standard, vintage framework, after 25 years. Figure 14 shows the result of this exercise. To achieve the target, fossil fuel taxes need to rise by 50% more under the putty-clay framework. Because the fossil fuel tax takes time to be effective, the near-term carbon stock is relatively high, but converges to a lower level in the long-term as the putty-clay dynamics fade. The economic effects of achieving this

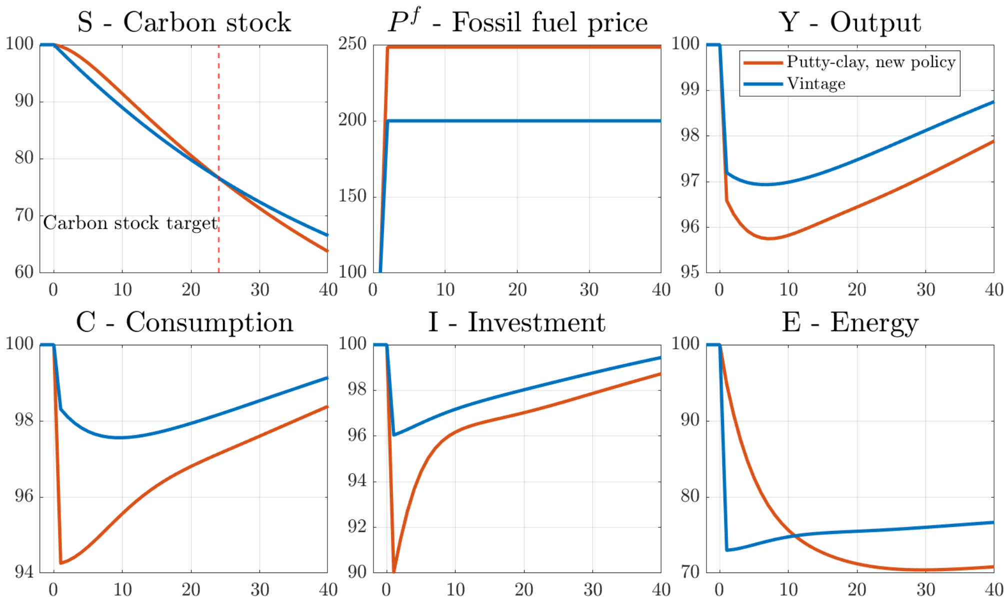


Figure 14: Targeting 2050 carbon stocks

target are much more damaging. Output declines more in the short-term and throughout the transition, under the putty-clay model. Consumption declines by nearly 2pp more in the short-term, while investment also declines more sharply and consistently.

4.5 Investment taxes and subsidies

In Appendix Sections A and B, we contrast the carbon tax with taxes and subsidies on investment in the dirty and green sector.[6] In contrast to the fossil fuel tax, the dirty investment tax has no effect on fuel intensity and works entirely through a reallocation of investment and production towards green energy and away from dirty energy. Similarly, the green energy investment subsidy also has no effect on fossil-fuel intensity and hence works entirely through the reallocation of investment and production towards the green sector. Among these two policies, the green energy subsidy is more effective in reducing fossil fuel use than the dirty investment tax. This reflects the fact that the capital share in the green energy sector is larger so that the investment subsidy leads to a greater reallocation than the dirty energy tax.

As a result of this, the carbon tax much more effective than investment taxes and subsidies in reducing fossil fuel use and therefore reducing the carbon stock. Nonetheless, the putty-clay nature of production implies that the transition in fuel usage in response to such a tax is far more gradual than one would obtain from a standard vintage capital model. This gradual adjustment occurs because of the irreversible nature of the fossil-fuel intensity embodied in the existing capital stock. This gradual adjustment also occurs because the tax on fossil fuel leads to greater investment in both the green and dirty energy sectors which raises the

[6]These could also approximate the effects of ESG investment strategies, to the extent that such approaches might the required return on investment heterogeneous by sector.

overall demand for energy in the short run.

5 Conclusion

We propose a multi-sector integrated assessment model to understand the economic impacts of a green transition. The key feature of our model is the use of putty-clay technology in each of the sectors. This allows us to model the fixity and irreversibility of capital, producing a vintage structure and potential underutilisation of old capital. These features of the capital composition are consistent with observations from the data on the energy sector in the US.

Using this model, we compare different policies to achieve a transition away from fossil fuel usage; green technology improvements, carbon taxes and investment taxes and subsidies. We find that carbon taxes are highly effective; as they specifically target fossil fuel usage rather than dirty energy production more generally, our calibrated model results suggest that these would reduce fossil fuel usage by 60%, more than other strategies. However, carbon taxes are also more costly and slower to be effective under a putty-clay framework. This is because pre-existing, sunk investments in highly fossil fuel dependent capital must be underutilised. The dirty energy capital stock which does continue to be used continues to emit heavily. To achieve the same carbon stock targets as under the vintage model, carbon taxes may need to be as much as 50% higher, resulting in a larger and persistent decline in output.

To compare with carbon taxes, we also simulate the effects of a green technology improvement (calibrated to match recent improvements in solar power technology) and taxes and subsidies on energy sector investment, of a similar order of magnitude to the carbon tax. Green technology improvements increase output, consumption and investment in the long-term, reducing fossil fuel usage by over 30%. Taxes on dirty investment and subsidies on green investment are less effective in achieving climate goals, as they don't lead the dirty energy sector to become more fuel efficient, while causing a misallocation of capital within the energy sectors.

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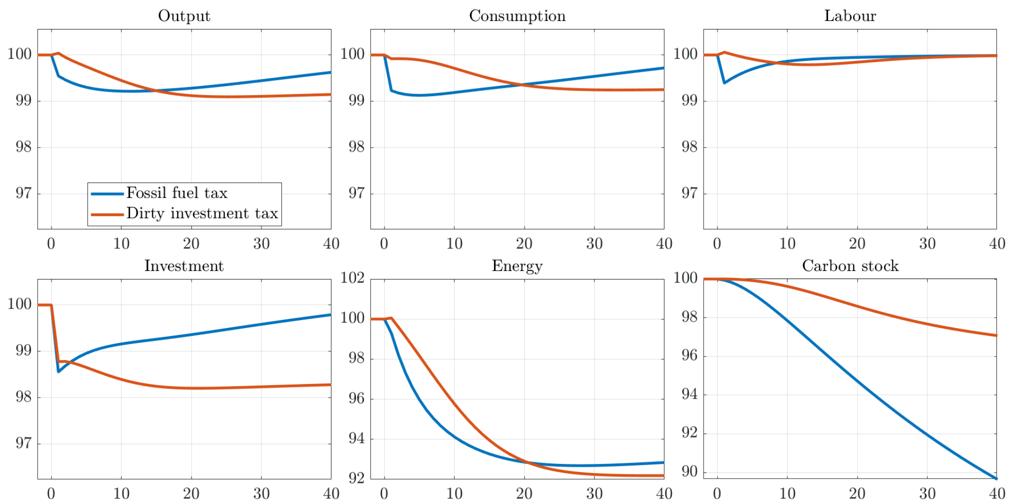
A Dirty Investment Tax

In this experiment we study the effect of a tax on investment in the dirty sector, which we compare to the effect of a 20% tax on fossil fuel. The investment tax is calibrated to generate the same revenue (in the initial steady state) as the fossil fuel tax. Figure A.1 displays the results of this experiment. The responses for the dirty investment tax are reported in red. The response to a fossil fuel tax of comparable magnitude are reported in blue.

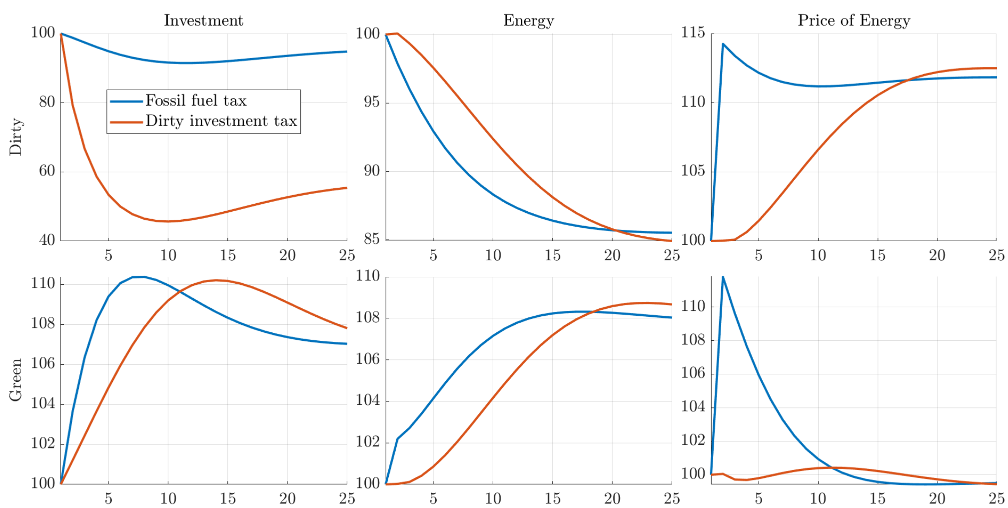
As can be seen in Panel A of Figure A.1, aggregate output, investment, energy and labour all decline gradually in response to the dirty investment tax. Aggregate output falls by slightly under one percent while aggregate investment falls by nearly two percent. The total quantity of energy falls by eight percent. While the drop in energy is comparable to that obtained from the fossil-fuel tax, the decline in the carbon stock is noticeably lower – three percent in the case of the dirty energy tax versus ten percent in the case of the fossil fuel tax.

Panel B of Figure A.1 highlights three key results. First, although the switch from dirty to clean energy is more rapid in the case of the fossil fuel tax, both taxes result in the same decline in dirty energy production and expansion in clean energy production in the long run. In addition, the dirty investment tax results in a substantial decline in investment in the dirty energy sector. In the long-run, dirty energy investment declines by over forty percent. In contrast, there is almost no decline in investment in response to the fossil fuel tax. Both taxes lead to roughly equivalent expansions in green energy investment.

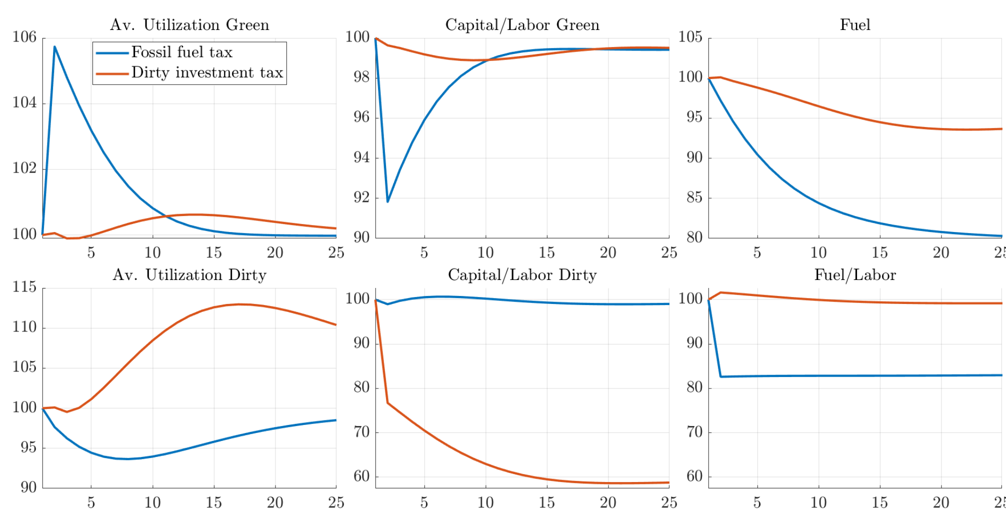
Panel C of Figure A.1 highlights the key distinction in these two taxes. The fossil fuel tax leaves the capital-labour ratio on new machines unchanged but causes a sharp decline in the fuel-to-labour ratio. The fossil fuel tax reduces fuel usage by causing production to switch from dirty to clean energy and by building a new stock of dirty-energy machines that are less fuel intensive. In contrast, in response to the dirty investment tax, there is no incentive to change the fuel-to-labour ratio since the relative costs of these variable inputs are unchanged by the investment tax. Moreover, utilization rates on dirty machines permanently rise by 10 percent as dirty energy producers save on capital by operating machines at a higher intensity. This combination of higher utilization rates and no change in fuel efficiency implies that total fuel usage only declines by six percent in response to a dirty energy tax despite the large drop in investment in that sector. In contrast, fossil fuel usage declines by 20 percent in response to the fossil fuel tax even though there is no long-run change in overall investment in the dirty-energy sector.



(a) Panel A: Aggregate Responses



(b) Panel B: Energy Sector Response



(c) Panel C: Sectoral Decomposition in Putty-Clay Model

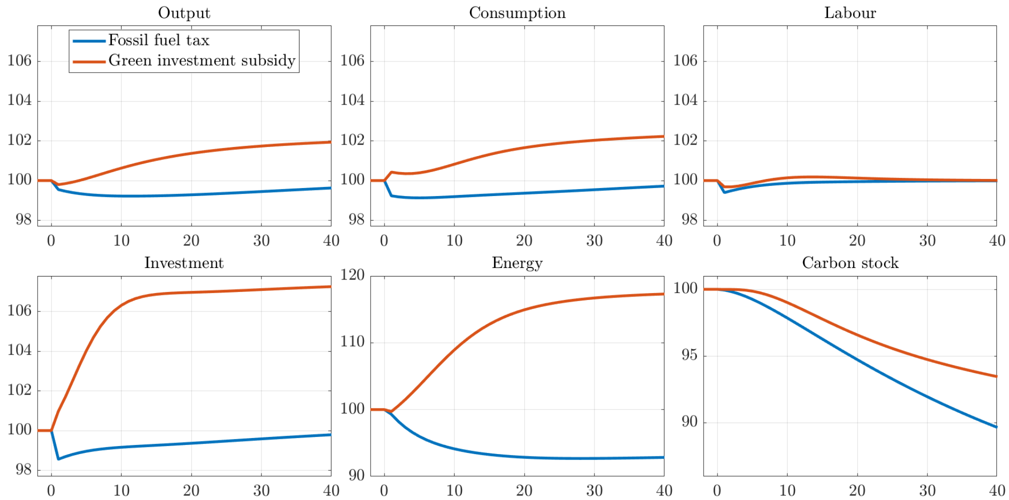
Figure A.1: Dirty Investment Tax

B Green Investment Subsidy

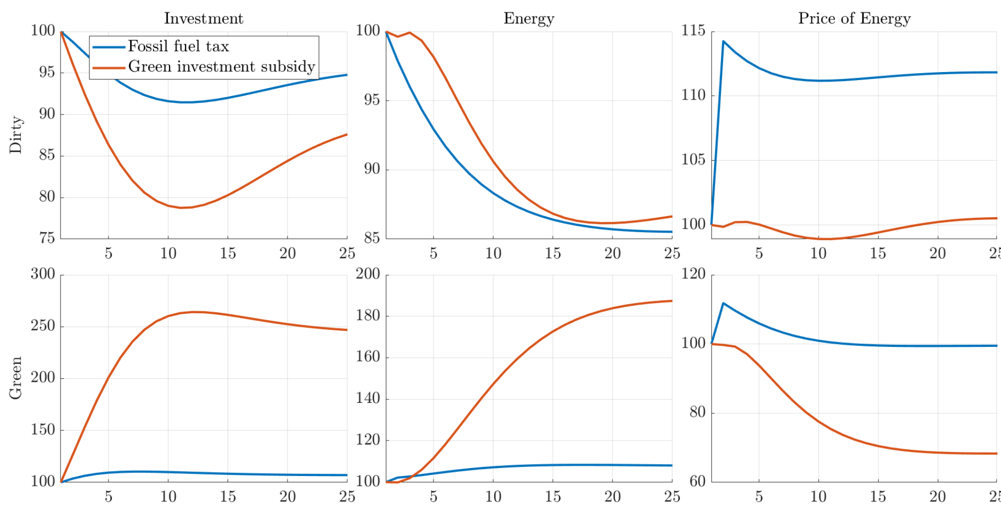
In this experiment we study the effect of a subsidy on investment in the green sector, which we again compare to the effect of a 20% fuel tax. As in the previous experiment, the investment subsidy is calibrated so that the cost is the same as the revenue generated by the fossil fuel tax in the initial steady state. The results of this experiment are shown in Figure B.1

The green energy subsidy causes an expansion in aggregate output, investment, labor and energy. These effects are sizeable – aggregate output rises four percent, aggregate investment rises seven percent and energy production increases nearly 20 percent in response to the subsidy. The green energy subsidy also causes a reallocation of energy production away from the dirty energy sector which falls by fifteen percent towards the green energy sector which expands by nearly 100 percent. Notably, this subsidy has no effect on the capital-intensity and fuel-intensity of new machines produced in the dirty-energy sector. Hence, the fifteen percent drop in dirty sector energy also implies a fifteen percent drop in fuel usage.

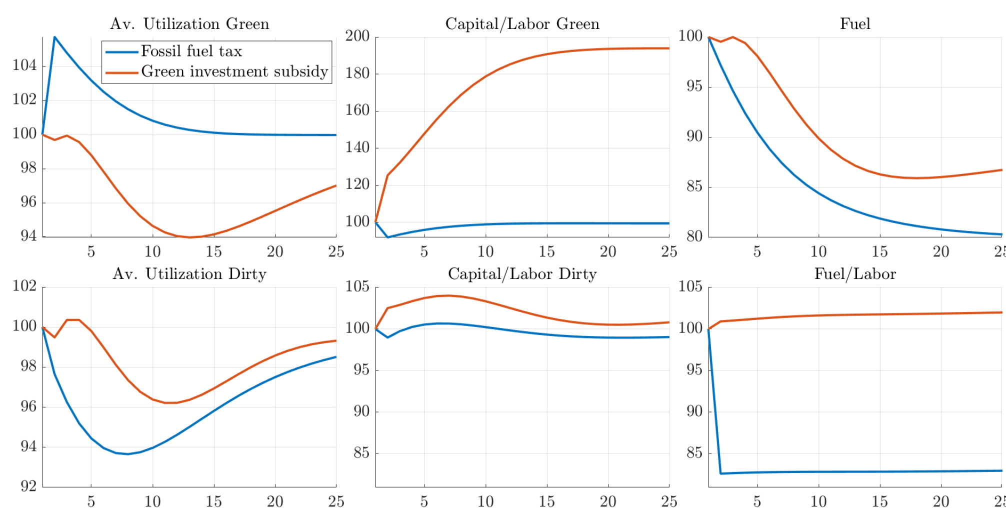
In contrast, the fossil fuel tax causes modest contractions in aggregate output, investment, and energy production. It also implies both a more rapid and larger overall decline in fuel usage. There are two important distinctions between these two policies. First, the green energy subsidy causes an expansion in investment and output. This leads to a rise in overall production and therefore fossil fuel usage in the short-run. Second, the green energy subsidy works entirely by reallocating production towards the green energy sector and away from the dirty energy sector whereas the fossil fuel tax also leads to a less-fuel intensive dirty energy sector. The combination of these forces implies that fossil fuel usage and carbon stocks fall by more in response to the fossil fuel tax relative to the green energy investment subsidy.



(a) Panel A: Aggregate Responses



(b) Panel B: Energy Sector Response



(c) Panel C: Sectoral Decomposition in Putty-Clay Model

Figure B.1: Green Investment Subsidy